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A PROBABILISTIC MODEL FOR RESOURCE ALLOCATION TO REDUCE PROJECT DURATION

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Abstract - Resource allocation is a significant component of the strategic planning of a project. Scarce resources are usually allocated among the different activities of the project for achieving the project goals. In simple projects, manual approaches can be used, but in more complex projects, algorithmic approaches are required in order to achieve the required goals. A project is considered successful if it meets the stakeholders' requirements within the given budget and is delivered within the planned schedule. In this paper we propose a model for resource allocation that is based on the correlation between the allocated resources for each activity in the project, and the probability of the project to achieve its planned goals within the expected schedule. The model considers serial activities where resources that are allocated to a specific activity cannot be transferred to another activity. Simulation results, presented in the paper, indicate that resources should be allocated among all project's activities such that the expected durations are evenly spread among all the activities.

Keywords - Project Management, Project Planning, Resource Allocation, Expected Duration.

I. INTRODUCTION

Resource-constrained project scheduling problem (RCPSP) is a well-known topic in project planning, and many models have been developed for static deterministic environments (Brucker et al., 1999; Herroelen et al.,1998; Kolisch and Padman, 2001). In these type of problems, limited resources are assigned to the project's activities, where each activity has an expected duration and resource requirements. The aim is to find an optimal assignment schedule in order to achieve a minimal duration for the entire project, subject to the precedence relations among the activities, the resource requirements of each activity and the available resources.

The solution provided by the RCPSP model is considered a preschedule or baseline schedule only, as project activities are commonly subject to considerable uncertainty during the execution stage, which often leads to schedule disruptions and project slippage. The uncertainties during project execution can stem from various sources such as human and/or machine failures, external disturbances etc.

Adlakha and Kulkarni (1989) present a bibliography of early research work (1966-1987) on stochastic Project Evaluation and Review Technique (PERT) networks, ranging from the classical PERT problem through exact analysis, approximation and bounds, and Monte Carlo sampling that deal with stochastic activity networks. Igelmund and Radermacher (1983a and 1983b) consider the optimization problem for stochastic project networks under resource constraints. In these projects, the activities' durations are given by probability distributions and the aim is to minimize the expected overall project duration. They propose a broad class of strategies, named "pre-selective" strategies. Stork (2001) developed theory and algorithms for better understanding of stochastic resource-constrained project scheduling problems. Starting from the observation that the widely used class of priority policies is not a good choice in terms of a 'robust' execution of projects, his work focusses on the representation of resource constraints by minimal forbidden sets, which are necessary for defining and handling pre-selective policies. An effective algorithm constructs all minimal forbidden sets of a given instance in a considerably compact representation. More recent research include Ratajczak-Ropel (2018), Kadri et al, (2018) and Bruni et al, (2018).

In this paper we present a new model for project planning that considers the probability distributions of each project activity, and their effect on the entire project duration. The probability of a successful completion of an activity in a specific duration is related to the resources allocated to that activity, and therefore the model is suitable for resource allocation plan. The model considers serial activities where resources that are allocated to a specific activity cannot be transferred to another activity.

II. PROBLEM FORMULATION

Consider a project that consists of n different and sequential activities. Each activity i is characterized by an expected duration $E(d_i)$ that is related to the amount of resources allocated to that activity – r_i, such that $d_i = F(r_i)$. Assuming the project's total amount of resources is limited, and that a resource that is allocated to a specific activity cannot be transferred to a different activity, the goal is to assign the resources to all activities such that the duration of the entire project is minimized. Following is the mathematical formulation of the problem:

$$min\{\sum_{i=1}^{n} E(d_i)\} \quad (1)$$

s.t.

$$d_i = F(r_i) \quad (2)$$

$$\sum_{i=1}^{n} r_i = C \quad (3)$$

The effect of the resource allocation on the expected duration of the activities (given by F) is determined in a probabilistic manner according to the specific resource and activity. For example, if the resource reflects the financial investment in the activity (e.g. in \$), the expected duration may be of exponential distribution, similar to the one shown in Figure 1a. If, on the other hand, the resource reflects man-power, the expected duration might be represented by normal distribution as shown in Figure 1b. In Figure 1a the expected duration is given by

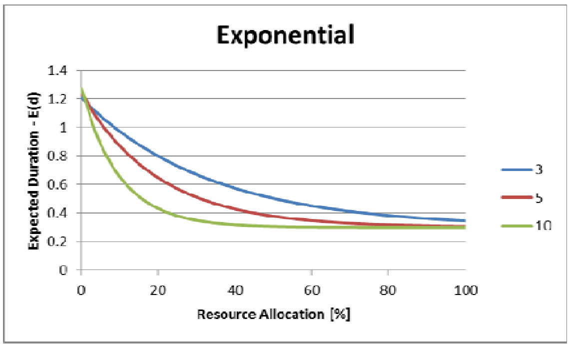
$$d_i = \lambda e^{-\lambda r_i} + C_i \quad (4)$$

where d_i is the expected duration of activity i, r_i is the resource allocated to activity i C_i is the minimal expected duration and $\lambda$is the activity constant.

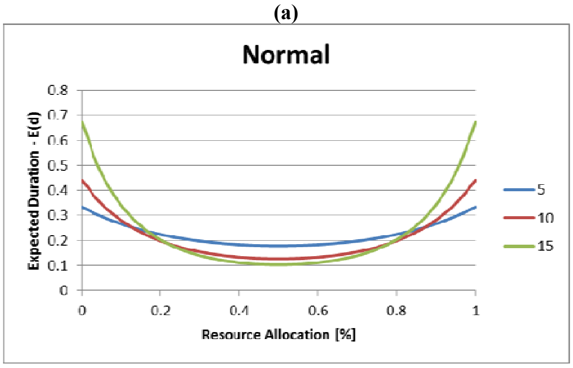
In Figure 1b the expected duration is given by

$$d_i = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r_i-\mu)^2}{2\sigma^2}} + C_i \quad (5)$$

where μ and σ^2 are the mean and STD respectively of the normal distribution.



(a)



(b)

Figure 1: The effect of resource allocation on the expected activity duration for exponential (a) and normal (b) distributions.

III. EXPECTED DURATION OF THE ENTIRE PROJECT

Following Eq. 1, the duration of the entire project d_p is given by

$$d_p = \sum_{i=1}^{n} d_i \quad (6)$$

where d_i is the duration of activity i. Since the expected value operator is linear, and since the duration of the entire project is the sum of all the expected durations of the project's activities (assuming no parallel activities are allowed), the expected duration of the entire project is given by

$$E(d_p) = E(\textstyle\sum_{i=1}^{n} d_i) = \sum_{i=1}^{n} E(d_i) \quad (7)$$

Given that the expected duration of each activity is determined by the amount of resources allocated to that activity (see Eq. 2), the expected duration is therefore determined by the resource allocation of all activities in the project. In the following section we provide a few illustrative projects that exemplify the effect of the resource allocation on the expected project duration.

IV. ILLUSTRATIVE EXAMPLES

Consider a project that consists of two activities (A and B). The total amount of resources for the entire project is 10. Table 1 shows all 9 possible combinations of resource allocations between the two activities. In this example we assume that the two activities are identical in terms of the expected duration based on the resource allocation for each activity.

Table 1: All combinations for resource allocations between two activities

Activity	Resource allocations								
A	1	2	3	4	5	6	7	8	9
B	9	8	7	6	5	4	3	2	1

Assume that the expected duration of each activity is given by the exponential distribution

$$E(d_i) = \frac{\lambda_i e^{-\lambda_i r_i}}{(\lambda_i + C_i)} + C_i \quad (8)$$

where r_i is the resource allocation for activity i, λ_i is the distribution rate and C_i is the distribution limit (the minimal expected duration) for activity i. Figure 2 illustrates the expected duration of one activity for $\lambda_i = 5$ and $C_i = 5$.

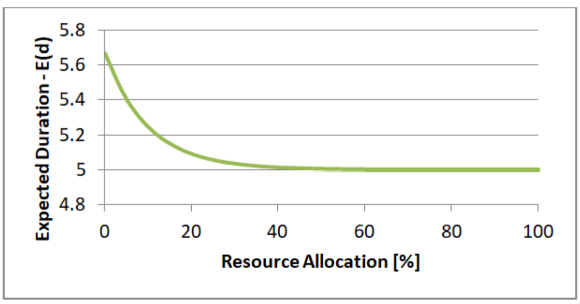


Figure 2: Expected duration for a single activity with exponential distribution where $\lambda_i = 5$ and $C_i = 5$.

Figure 3 shows the expected duration of the entire project (activities A and B) for the different combinations of resource allocations (the figure shows the portion of resource allocated to activity A). As shown, the minimal expected duration is for allocation of 50% of the total resources for activity A, and the maximal expected entire project duration is for the extreme allocations where 10% or 90% of the resources are allocated to one activity.

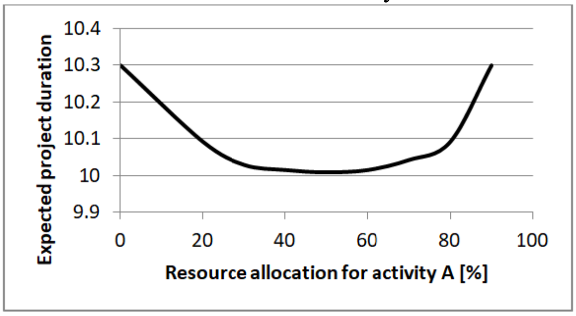
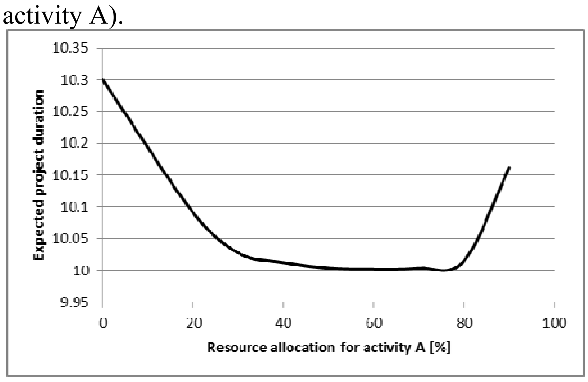
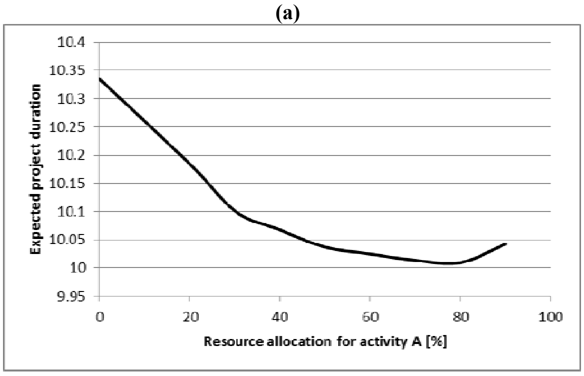


Figure 3: Expected duration of the entire project for different resource allocations

Next, Figure 4 shows the expected project duration for cases where the two activities are different. In Figure 4a, $\lambda_A = 20$, $C_A = 5$, $\lambda_B = 10$ and $C_B = 5$. As shown, the expected duration of the entire project is not symmetric as is the case in Figure 3. The minimal expected duration is for allocating 60% of the resources to activity A, and the maximal expected duration is for allocating 10% of the resources to activity A. In a different scenario, where $\lambda_A = 40$, $C_A = 5$, $\lambda_B = 5$ and $C_B = 5$ (Figure 4b), the minimal expected duration is for allocating 80% of the resources to activity A (the maximal expected duration is again for allocation of 10% of resources to activity A).



(a)



(b)

Figure 4: Expected project duration for exponential expected duration of the activities where (a) $\lambda_A = 20$, $C_A = 5$, $\lambda_B = 10$ and $C_B = 5$ and for (b) $\lambda_A = 40$, $C_A = 5$, $\lambda_B = 5$ and $C_B = 5$.

Finally, Figure 5 shows the expected project duration for the case where the expected duration for the first activity has an exponential distribution (Eq. 8) with $\lambda_A = 10$, $C_A = 5$, and the expected duration of activity B has a normal distribution in the form of Eq. 5 with $\mu = 5$ and $\sigma^2 = 5$. In this case the minimal expected duration is for allocating 50% of the resources to activity A.

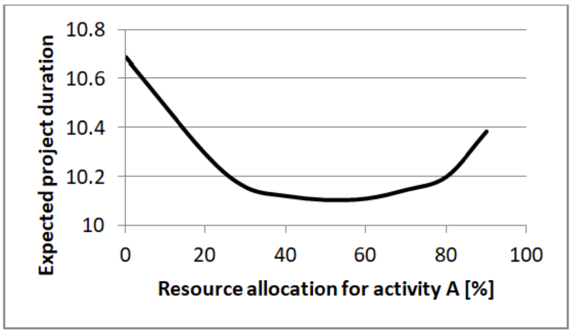


Figure 5: The expected project duration for exponential distribution of activity A (λ_A=10, C_A=5), and normal distribution for activity B ($\mu = 5$ and $\sigma^2 = 5$).

CONCLUSIONS

In this paper we present a model for resource allocation to a project's activities. The resource allocated to a specific activity affects the expected duration of that activity. Given limited resources, the model constructs the expected duration of the entire project using the linearity of the expected value operator. Initial simulation results show that resource allocation should consider the probabilistic distribution of the expected duration of each activity in the project. As stated, the results presented in this paper are preliminary, and further analysis is required in order to determine an analytical solution for this type of the resource-constrained project scheduling problem (RCPSP).

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