Abstract— Vehicle Bridge Interaction (VBI)-a coupled, non-linear time dependent dynamic analysis for a single span bridge was carried out in the present study. Two different values of span length i.e. 60 m and 80 m were chosen for VBI analysis. The effects of VBI were studied for four different vehicular speeds viz. 50 Km/hr, 70 Km/hr, 100Km/hr and 120 Km/hr. A two dimensional vehicle bridge model was developed to study the influence of bridge’s span on the VBI using mode superposition method. Euler-Bernoulli beam theory was adopted to develop the kinetic model of the bridge sub-system and the two-axle lumped vehicle model. Four degrees of freedom system with only one vehicle moving across the bridge was considered for the present study. Considering one vehicle with constant parameters moving across the bridge for each model, a comparative study was conducted between span 60m and 80m. The results revealed a crucial role of bridge spans in influencing the vehicle-bridge vibration.

Keywords— Euler-Bernoulli beam theory, Mode Superposition method, Vehicle Bridge Interaction (VBI), Vibration.

INTRODUCTION:

Due to rapid urbanization in the world the number of bridges has increased to wide number with increasing bridge span, vehicle speed so it becomes important to study vehicle bridge interaction. Vibration is an important phenomenon in a bridge, which may cause due to irregularities of bridges, acceleration, braking and weight of the vehicles. In the meanwhile, this vibration gets transferred to the pier of the bridge, so when a vehicle running with high speed crosses the bridge it may cause damage to the bridge structure. The vehicle bridge interaction is a coupled, non linear time dependent dynamic problem and as suggested by Yang et al. (2004) in comprehensive review of bridge vehicle interaction studies that before 1940s, the investigation on bridge dynamic responses were mainly performed with analytical or approximate solutions for simple and fundamental problems. Later on analysis with more realistic bridge and vehicle models became possible with digital computer. The dynamic responses of two-span beam models under moving load were developed by Ayre et al. (1950) and Ayre and Jacobsen (1950) and later Vellozzi (1967) studied the vibration of suspension bridges. The vibration occurs under the influence of vehicular movement on suspension bridge was investigated by Chatterjee et al. (1994) with the torsional vibration taken in account. Road surface or pavement roughness has a significant effect on the impact response of bridges (Paultry et al., 1992). In order to study the vehicle bridge interaction system two sets of equation of motion, one for the bridge and another for the vehicle. To solve the above mentioned equation of motion different iterative procedure been used (Hwang and Nowak, 1991; Green and Cebon, 1994; Yang and Fonder, 1996). For analysis the vehicle bridge interaction systems Lagrange’s equation with multipliers and constraint equation has been applied (Bleijwaset al., 1979). Varieties of methods has been employed to solve the second order differential equation of motion for vehicle bridge interaction problems which includes, (i) superposition method (Bleijwaset al.,1979; Wu and Dai,1987; Galdos et.al., 1993); (ii) Direct integration methods, such as Newmark-β method (1959) (Inbanathan and Wieland, 1987;Yang and Lin,1995), Wilson’s θ method (Sridharan and Malik,1979), and fourth order Runge-Kutta method (Chu et al., 1986). Nowadays high complexity vehicle and bridge model with wide numbers of degree of freedom to study the vehicle bridge interaction can be done using a simplified vehicle bridge model in order to identify the influence of relevant parameters on dynamic vehicle bridge system.

In the present study an attempt has been made to develop a simplified two dimensional vehicle-bridge model using MATLAB software based on Newmark-β method to study how the change in span length will influence the vehicle-bridge vibration. Vehicle model with single axle of two degree of freedom, double-axle two dimensional models with four degree of freedom are the most widely used models. In this study the two-axle lumped vehicle model with four degree of freedom is considered and the bridge is modeled using Euler-Bernoulli beam theory with only one vehicle on the bridge.

Governing Equations:

Vehicle model:

Double-axle two dimensional vehicle model with four degree of freedom is considered in this study. Figure 1 is a model with four degree of freedom in which the body of the vehicle has two degrees of freedom(up and down of freedom, \(x_3\) and node degree of freedom \(\theta\)). The front and back wheel set have a vertical displacement degree freedom of \(x_1\) and \(x_2\) separately. Thus, the kinetic equilibrium function of the vehicle about every degree of freedom according to Newton’s law.
Body vibrates up and down:
\[ m_1 \ddot{y}_1 + c_1 \left( \dot{y}_1 - \dot{y}_{11} + \dot{\theta}_1 \right) + c_2 \left( \dot{y}_1 - \dot{y}_{12} - \dot{\theta}_2 \right) + k_1 \left( y_1 - y_{11} + \theta_1 \right) + k_2 \left( y_1 - y_{12} - \theta_2 \right) = 0 \]  

(1.1)

Body nodes vibration:
\[ \begin{align*}
   &j_1 \left( \ddot{y}_1 - \dot{y}_{11} + \dot{\theta}_1 \right) - c_1 \left( \dot{y}_1 - \dot{y}_{11} + \dot{\theta}_1 \right) - c_2 \left( \dot{y}_1 - \dot{y}_{12} - \dot{\theta}_2 \right) \\
   &+ k_1 \left( y_1 - y_{11} + \theta_1 \right) + k_2 \left( y_1 - y_{12} - \theta_2 \right) = 0
\end{align*} \]  

(1.2)

Front axle vibrates in vertical motion:
\[ \begin{align*}
   &m_{11} \ddot{y}_{11} - c_1 \left( \dot{y}_{11} - \dot{y}_1 + \dot{\theta}_1 \right) - k_1 \left( y_{11} - y_1 + \theta_1 \right) \\
   &+ c_1 \left( \dot{y}_{12} - \dot{y}_{11} + \dot{\theta}_2 \right) + k_1 \left( y_{12} - y_{11} - \theta_1 \right) = 0
\end{align*} \]  

(1.3)

Back axle vibrates in vertical motion:
\[ \begin{align*}
   &m_{12} \ddot{y}_{12} - c_2 \left( \dot{y}_{12} - \dot{y}_1 - \dot{\theta}_2 \right) - k_2 \left( y_{12} - y_1 - \theta_2 \right) \\
   &+ c_2 \left( \dot{y}_{11} - \dot{y}_{12} - \dot{\theta}_2 \right) + k_2 \left( y_{11} - y_{12} + \theta_2 \right) = 0
\end{align*} \]  

(1.4)

\[ \text{Figure.1: Double-axle vehicle model with four degree of freedom} \]

Newton’s law is used to build this model and the equation is given in matrix form which is as follows:
\[ [M_v] \ddot{y}_v + [C_v] \dot{y}_v + [K_v] y_v = [F_v] \]  

(2)

Where, \([M_v]\) is mass matrix, \([C_v]\) is the damping matrix, \([K_v]\) is the stiffness matrix, \([y_v]\) is the matrix of the degree of freedom of the vehicle, \([F_v]\) is the matrix of exciting force of vibration of the vehicle.

**Bridge model:**
To develop the bridge model Euler Bernoulli theory is used. According to Euler-Bernoulli beam theory, the governing equation for flexural vibrations can be written as:
\[ EI \dddot{y}_b + \frac{\partial^2 y_b}{\partial x^2} + \rho \frac{\partial^2 y_b}{\partial t^2} + \mu \frac{\partial y_b}{\partial t} = -F(x,t) \delta(x-vt) \]  

(3)

**II. METHODOLOGY**
To develop the vehicle bridge model first of all the two axle forces acting on the bridge is to be known. The eqn. (2) which is for the double-axle two dimensional four degree of freedom is to be calculated out in order to get the two contact axle forces acting on the bridge. These forces thus obtained will be used to model the bridge based on Euler Bernoulli theory. As mentioned above Eqn. (2) is flexural vibration equation for one contact load but in our study we need to design it for two contact load, so the Eqn. (2) will know be written as:
\[ EI \dddot{y}_b + \frac{\partial^2 y_b}{\partial x^2} + \rho \frac{\partial^2 y_b}{\partial t^2} + \mu \frac{\partial y_b}{\partial t} = -\sum_{i=1}^2 F_i(x,t) \delta(x_i-vt) \]  

(5)

In this study it has been assumed that the vibration of the bridge is mainly made up by basic modes of vibration with lower frequency (Yang, J. and Duan, R., 2013). As it is has been considered that using the lower mode of vibration is of high accuracy to describe the vibration of bridge structure (Clough, R.W. et al., 1993). The degree of freedom of the bridge’s structure can be decrease by using the mode superposition method. According to the mode superposition method we can have:
\[ y_b(x,t) = \sum_{i=1}^K \varphi_i(x) \eta_i(t) \]  

(6)

Here, the mode superposition method is combined with the Euler-Bernoulli beam theory to get the bridge equation. Thus, the equation (4) will be written in mode based bridge equation as:
\[ i \ddot{\eta}_n + 2 \zeta \omega_i \dot{\eta}_n + \omega_i^2 \eta_n = -F_1(t) \varphi_{i1} \delta_1 - F_2(t) \varphi_{i2} \delta_2 \]  

(7)

**Construction of the model:**
The complete model of double-axle vehicle-bridge is developed as shown in the figure (2). In this study the model is develop to analyze the vehicle bridge interaction under three time ranges.

\[ \text{Figure.2: Complete model of vehicle-bridge model} \]
Considering the irregularities of the bridge’s surface, the system model of the vehicle-bridge as

\[ M(t) \ddot{Y} + C(t) \dot{Y} + K(t)Y = Q(t) \]  

(8)

Where, 

\[ [M(t) \quad [C(t) \quad [K(t)] \] are respectively (n+4) orders mass matrix, damping matrix and stiffness matrix. \( Q(t) \) is also (n+4) orders vector matrix, \( Y \) is (n+4) order free vector.

Then we get,

\[ Y = \begin{bmatrix} y_1 \quad \theta_1 \quad y_2 \quad \eta_1 \quad \eta_2 \quad \ldots \quad \eta_{n-1} \end{bmatrix}^T \]  

(9)

This is the displacement vector of the system and the matrix are give

\[ \Phi(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \]

This is the displacement vector of the system and the matrix are give

\[ K(t) = \begin{bmatrix} k_{11} & -k_{21} & k_{31} & \cdots & \cdots & \cdots & \cdots \\ -k_{12} & k_{22} & -k_{32} & \cdots & \cdots & \cdots & \cdots \\ k_{13} & -k_{23} & k_{33} & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ k_{1n} & -k_{2n} & k_{3n} & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \]

(7)

Simulation using MATLAB:

In this study the governing equation as mentioned before will be simulated using the MATLAB software. The equations are numerically solved based on Newmark-Beta method. The parameters for the bridge and the vehicles which are been used for the simulations were taken from Zhu, X. and Hao, H. (2012). Table.1 parameter of the vehicle and Table.2 is the parameter of the bridge that is considered as an input variable in this present study.

<table>
<thead>
<tr>
<th>Table.(1): Vehicle parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
</tr>
<tr>
<td>( m_1 / kg )</td>
</tr>
<tr>
<td>( J / (kg.m^2) )</td>
</tr>
<tr>
<td>( m_2 / kg )</td>
</tr>
<tr>
<td>( m_3 / kg )</td>
</tr>
<tr>
<td>( k_{13} / (N.m^{-1}) )</td>
</tr>
<tr>
<td>( k_{14} / (N.m^{-1}) )</td>
</tr>
<tr>
<td>( v / (km.h^{-1}) )</td>
</tr>
</tbody>
</table>

Table.(2): Bridge parameters

<table>
<thead>
<tr>
<th>The span length / (m)</th>
<th>The mass of bridge / kg</th>
<th>The Structural rigidity / (N.m^2)</th>
<th>The damping coefficient per unit Length / ( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>2.79 X 10^6</td>
<td>1.15 X 10^12</td>
<td>0.05</td>
</tr>
<tr>
<td>80</td>
<td>3.23 X 17</td>
<td>2.7 X 10^14</td>
<td>0.03</td>
</tr>
</tbody>
</table>

III. RESULTS AND DISCUSSION

In this study two vehicle bridge models are developed for span length 60 m and 80m on considering only one vehicle moving across the bridge. The results thus obtained for different speed of the vehicle are discussed below.

The eqn. (7) will be considered for three time range firstly, when the vehicle starts to run on the bridge with time range \([0 \leq t < \frac{\alpha}{v}]\) and we get \( \delta_1 = 1 \), \( \delta_2 = 0 \). Secondly, when the vehicle is running on the bridge with time range \([\frac{\alpha}{v} \leq t < \frac{\beta}{v}]\) and we get \( \delta_1 = 0 \), \( \delta_2 = 1 \) and lastly, when the vehicle running down the bridge just on the right edge the time range is \([\frac{\alpha}{v} \leq t < \frac{\beta}{v}]\) and we get \( \delta_1 = 0 \), \( \delta_2 = 1 \).
Dynamic Impact Analysis of Vehicle Bridge Interaction System

**FIGURE 3:** The vertical displacement of axles at different location for different vehicular speed.

(a) bridge span 60 m and (b) for bridge span 80 m.

From figure 3 it has been found that for both the span length the vertical displacement of the axle is maximum at the front and back of mid span. Thus, the vertical displacement of the axle will be minimum at the mid span for any span length.

**FIGURE 4:** The dynamic displacement of the vehicle on the first contact point in different vehicular speed.

(a) for span length 60 m and (b) for span length 80 m.

Figure 4 gives the nodal dynamic displacement of the vehicle at different vehicular speed for span length 60 m and 80 m. If an average line is considered we can have that with increase in span length there is an increase in nodal dynamic displacement of the vehicle for same speed.

**FIGURE 5:** The dynamic displacement of the bridge on the first contact and second point in different vehicular speed.

(a) and (c) for span length 60 m and (b) and (d) for span length 80 m.

From figure 5 it is clear that for both the span length the maximum displacement of the bridge happens when the vehicle locates in the front and back of the mid span. Now, considering the first contact point and second contact point we have bigger the span of the bridge, the maximum displacement will be bigger for same speed of vehicle.
CONCLUSION

In the present study a two dimensional vehicle-bridge model is developed and the influence of bridge's span on the vehicle-bridge vibration is studied using mode superposition method. Euler-Bernoulli beam theory is used to develop the kinetic model of the bridge sub-system and two-axle lumped vehicle model with four degree of freedom is considered. Only one vehicle with constant parameters is considered moving across the bridge during the study. For simulating this model, MATLAB software based on Newmark-β method is used.

It has been observed from the simulation result that for same speed of the vehicle there will be maximum vertical displacement of axle at the mid span, and for nodal dynamic displacement of vehicle it will increase with increase in span length. For bridge displacement, the maximum displacement happens when the vehicle is located in the front or back of the mid span and considering the contact point, for vehicle moving with same speed, bigger the span length, maximum will be the displacement.

REFERENCES:


