FINITE ELEMENT DYNAMIC ANALYSIS OF THIN PLATES WITH COMPLICATED GEOMETRY

[1]SALEEMA PANDA, [2]MANORANJAN BARIK

[1,2]Department of Civil Engineering,
National Institute of Technology, Rourkela, Odisha 769008, India
E-mail: [1]saleema.panda@gmail.com, [2]manoranjanbarik@yahoo.co.in

Abstract— The plates used as structural elements in the field of aerospace, naval and civil engineering take different shapes due to their functional and structural requirements as well as aesthetic considerations. In this paper, finite element dynamic analysis of different complicated shaped thin plates is presented. In the formulation, the arbitrary planform of the whole plate is mapped into a square domain where a cubic serendipity shape function is used to represent the complicated geometry and an ACM plate bending element is considered for the displacement function. This element is capable to model different geometries just like isoparametric element without the shear locking problem and generation of spurious mechanisms which is inherent in the isoparametric formulation.

Keyword— Plate Bending, Finite Element Method, Thin Plate, Dynamic Analysis.

I. INTRODUCTION

The functional and structural requirements along with the aesthetic considerations demand the shapes of the plates to be complicated when they are used as structural elements in the field of aerospace, naval and civil engineering. Their behavior must be known under dynamic conditions. In the literature, the studies carried out are mostly on plates of regular geometries and those on complicated shapes are scanty. Therefore, dynamic analysis of different complicated shaped plates is presented in this paper. Earlier, Huang (1990) proposed a mixed FE-BE method for analyzing coupled vibration of complicated floating plates. A consistent boundary element approach is presented by El-Zafrany et al. (1994) for bending analysis of arbitary shaped thin plates. Bending analysis of thin plates is done by Hamouche et al. (1997) with spectral solution methodology. A mesh free method is formulated by Liu and Chen (2001) to find static deflection and natural frequency of thin plates and solved hexagon, elliptical and a plate with a hole of complicated shape. Karami and Malekzadeh (2002, 2003) employed differential quadrature methodology for quadrilateral plates and Shi et al. (2009) proposed boundary knot method for free vibration of arbitrary shaped plates. Green quasifunction method is considered by Li and Yuan (2012) for free vibration analysis of clamped thin plates. Jonckheere et al. (2015) studied wave based method for dynamic bending analysis of quadrilateral plates.
Kirchoff plate theory which is meant for thin plates cannot be used for arbitrary geometry plates and Mindlin plate elements when applied to thin plates face shear locking problems. So, Barik and Mukhopadhyay (1998) have developed a rectangular element through which any arbitrary shaped thin plates can be analyzed. In this paper, the same formulation is applied for the analysis of complicated shaped thin plates. Its usefulness, efficiency and generality are shown by presenting numerical examples.

II. MAPPING OF THE PLATE

The arbitrary shape of the plate is mapped (Barik and Mukhopadhyay, 1998) approximately into a [-1, +1] region in the ξ-η plane with the help of the cubic serendipity shape function (Zienkiewich and Taylor, 1989). The mapped square plate is now discretized into a number of elements and each element is being mapped with the same cubic serendipity shape function to a natural coordinate element of domain [-1, +1].

III. DISPLACEMENT INTERPOLATION FUNCTION

For the proposed element, the four-noded rectangular ACM plate bending element with three degrees of freedom (w, Θx, Θy) at each node is considered. The interpolation functions for the bending are the usual ones presented in detail in Barik and Mukhopadhyay (1998).

IV. BOUNDARY CONDITIONS

As a general case, the stiffness matrix for a curved boundary supported on elastic springs continuously spread along the boundary line is used. Considering a local axis system x_1 - y_1 at a point on a curved boundary along the direction of the normal to the boundary at that point, the displacement components along it can be obtained. The details are in Barik and Mukhopadhyay (1998).

V. RESULTS AND DISCUSSION

Free vibration analysis for two different complicated shapes i.e. semi-circular semi-elliptical and diamond shaped are carried out with mesh divisions for the whole plate. Fundamental frequencies are obtained and corresponding mode shapes are also presented. The notations used for boundary conditions (BC) for all edges simply supported and clamped are SSSS and CCCC respectively.

A. Semi-circular Semi-elliptical Shaped Plate

A semi-circular semi-elliptical shaped plate in Fig. 1 is analyzed and its fundamental frequencies for SSSS and CCCC boundary conditions are tabulated in Table 1. Fig. 2 and 3 presents first four mode shapes of SSSS and CCCC plate respectively.

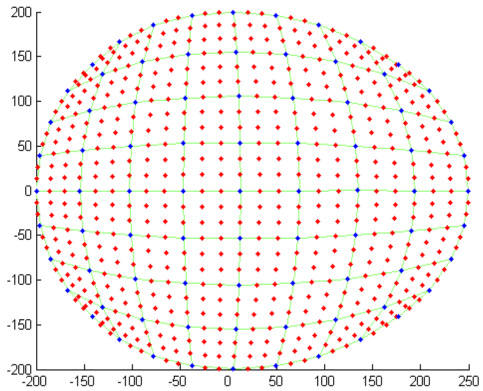
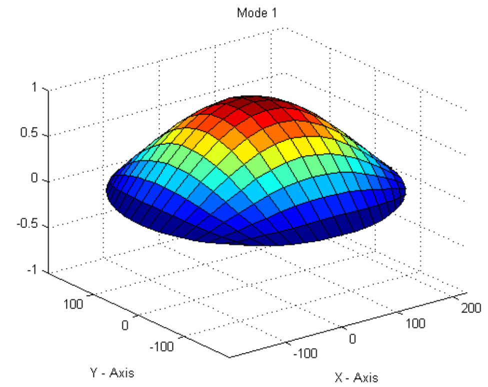
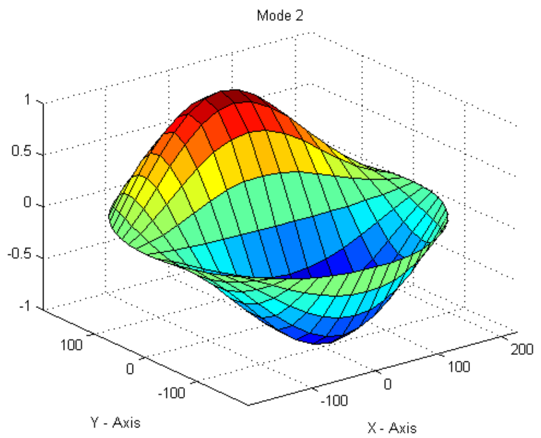
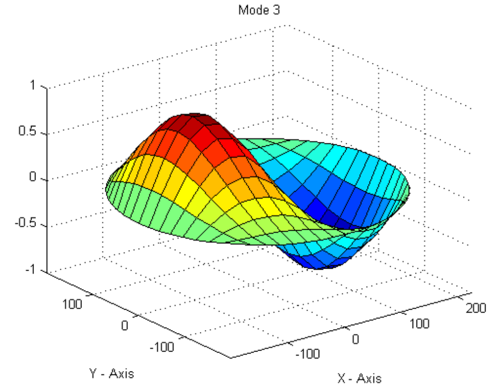


Fig. 1: A typical 8×8 mesh with boundary nodes of Diamond Shaped Plate







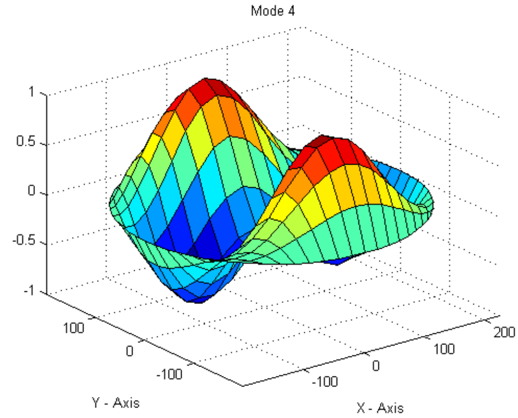


Fig. 2: Mode shapes of simply supported Semi-circular Semi-elliptical Plate

Table 1: Frequency parameter ($\omega' = \omega b^2(\rho h/D)^{1/2}$) for Semi-circular Semi-elliptical Plate

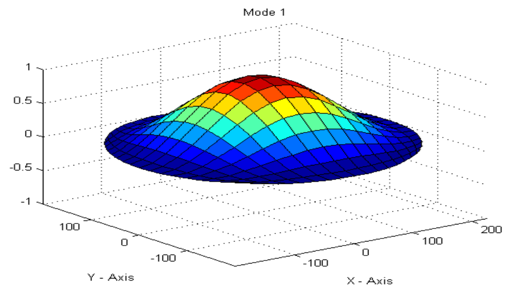
BC	Mode 1	Mode 2	Mode 3	Mode 4
SSSS	7.033	18.189	20.736	34.293
CCCC	14.359	27.992	31.456	46.716

1.1. Diamond Shaped Plate

Fig. 4 shows a typical diamond shaped plate which is analyzed and its fundamental frequencies for SSSS and CCCC boundary conditions are tabulated in Table 2. The first four mode shapes of SSSS and CCCC plate are presented in Fig. 5 and 6 respectively.

Table 2: Frequency parameter ($\omega' = \omega r^2(\rho h/D)^{1/2}$) for Diamond Shaped Plate

BC	Mode 1	Mode 2	Mode 3	Mode 4
SSSS	38.197	89.312	89.313	130.516
CCCC	66.869	133.468	133.469	185.230



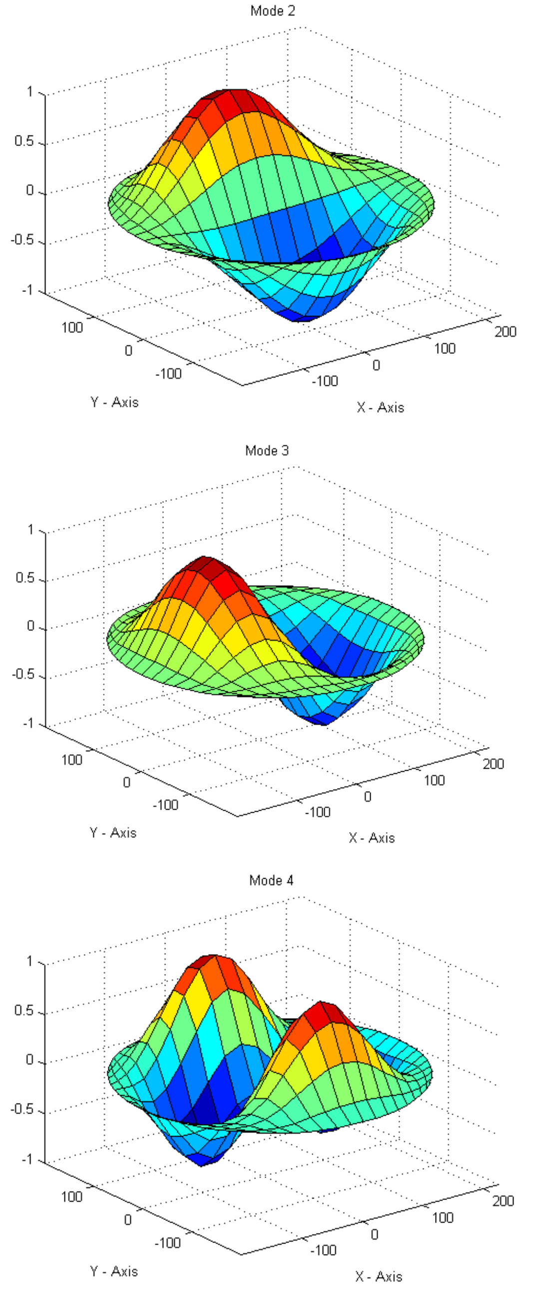


Fig. 3: Mode shapes of clamped Semi-circular Semi-elliptical Plate

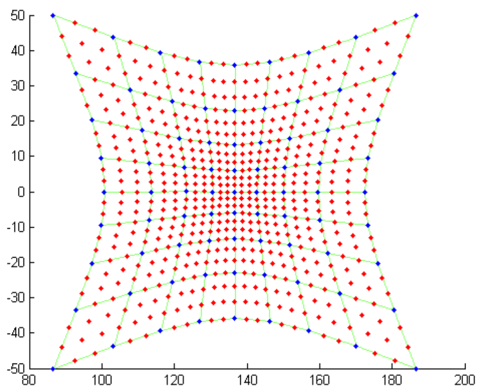


Fig. 4: A typical 8×8 mesh with boundary nodes of Diamond Shaped Plate

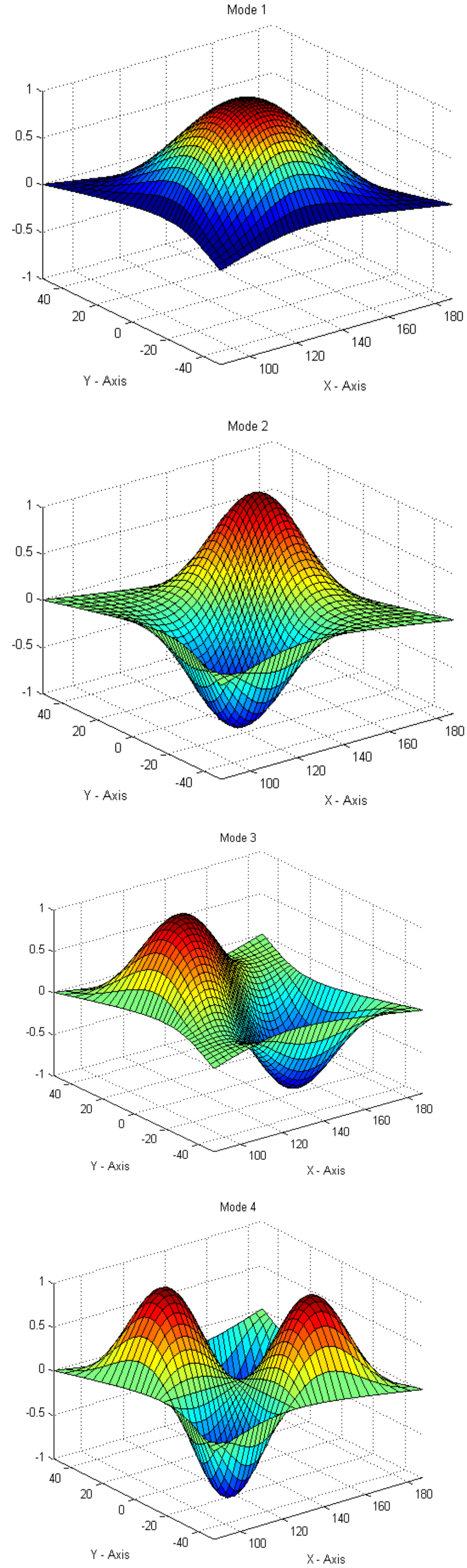


Fig. 5: Mode shapes of simply supported Diamond Shaped Plate

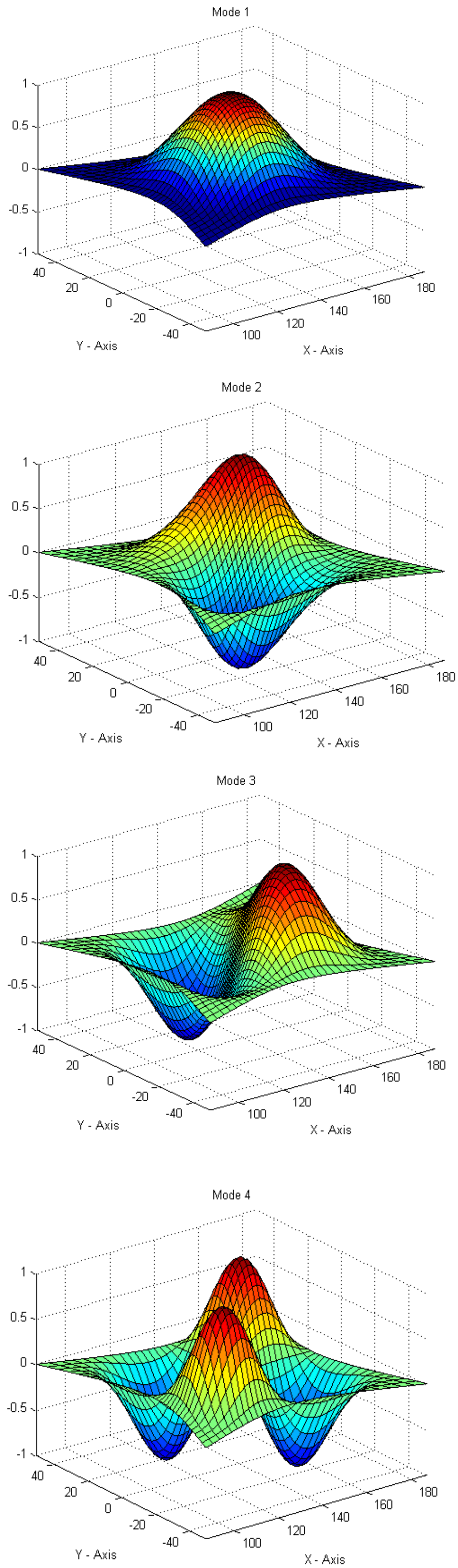


Fig. 6: Mode shapes of clamped Diamond Shaped Plate

CONCLUSIONS

In order to have the linear dynamic analysis of thin plate of arbitrary shape, the formulation is generalized by means of a mapping technique so that the analysis is performed in a square domain. Many researchers have used different elements to analyze plates but these elements are limited to solve a particular type of geometry only. In the present paper, the element has all the advantages of the isoparametric element to model an arbitrary plate shape and without the disadvantages of the shear locking problem etc. The versatility of the element is proved by undertaking different plate geometries. These are new results as no such geometries are analyzed in any previous published literatures.

REFERENCES

[1] Barik, M. & Mukhopadhyay, M. (1998). Finite element free flexural vibrational analysis of arbitrary plates. *Finite Element in Analysis and Design, 29*, 137-151.

[2] El-Zafrany, A., Debbih, M. & Fadhil, S. (1994). A modified Kirchhoff theory for boundary element bending analysis of thin plates. *InternationalJournal of Solids and Structures 31 (21),* 2885–2899.

[3] Hamouche, N.C., Warsi, Z.U.A. & McWhorter, J.C. (1997). A spectral solution for the bending of arbitrary shaped plates. *Computers & Structures, 63(1),* 159-171.

[4] Huang, Y.Y. (1990). A mixed FE-BE method for coupled vibrations of floating plates of complicated shape. *Appt. Math. Modelling, 14(2).*

[5] Jonckheere, S., Vandepitte, D. & Desmet, W. (2015). A wave based approach for the dynamic bending analysis of Kirchhoff plates under distributed deterministic and random excitation. *Computers & Structures, 156,* 42-57.

[6] Karami, G. & Malekzadeh, P. (2002). Static and stability analyses of arbitrary straight-sided quadrilateral thin plates by DQM. *International Journal of Solids and Structures, 39 (19)*, 4927–4947.

[7] Karami, G., Malekzadeh, P. (2003). An efficient differential quadrature methodology for free vibration analysis of arbitrary straight-sided quadrilateral thin plates. *Journal of Sound and Vibration 263 (2),* 415–442.

[8] Li, S. & Yuan, H. (2012). Green quasifunction method for free vibration of clamped thin plates. *Acta Mechanica Solida Sinica, 25 (1)*, 839–855.

[9] Liu, G.R., & Chen, X.L. (2001). A Mesh-free method for static and free vibration analyses of thin plates. *Journal of Sound and Vibration 241 (5),* 415–442.

[10] Shi, J., Chen, W. & Wang, C. (2009). Free Vibration Analysis of Arbitrary Shaped Plates by Boundary Knot Method. Acta Mechanica Solida Sinica, (4), 328-336.

[11] Zienkiewich, O.C. & Taylor, R.L., (1989). The finite element method, 4th Edition. McGraw-Hill.