ANALYSIS OF STUDENTS ERROR IN PROVING TRIGONOMETRIC IDENTITIES

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Abstract - The objective of this study is to describe kinds of error that were done by mathematics students of State Malang University in proving trigonometric identities based on Watson's criteria. This study is kind of qualitative research. The subject of this study are 33 mathematics students that contain 2 students with high mathematics skill, 3 students with medium mathematics skill, and 26 students with low mathematics skill.The data was obtained through ways of seeing the answer of students in complete three problems of proving trigonometric identities. The answers are analyzed descriptively to find out the student's error type. The results of this study are (1) Types of errors made by high mathematics skill subjects in proving trigonometric identities are improper procedures and skills hierarchy problems. (2) The types of errors made by medium mathematics skill subjects in proving trigonometric identities are improper procedures, indirect manipulations, skill hierarchy problems, and other categories; (3) The type of errorsmade by lowmathematics skill subjects in proving trigonometric identities are lost data, improper procedures, and indirect manipulation.

Keywords - Error analysis, Trigonometric Identities, and Watson criteria.

I. INTRODUCTION

According to Burton (Uzer, 2002) teaching in principle is to guide students in teaching and learning activities or contain the sense that teaching is an attempt to organize the environment in relation to learners and teaching materials that lead to the learning process. Burton's statement implies that lecturers can act as organizers of student learning activities. With regard to utilizing the environment in supporting teaching and learning activities, lecturers are responsible for being able to become facilitators in the learning process. In addition, lecturers are also responsible for evaluating the learning outcomes and progress of learning and make a careful diagnosis of the difficulties and needs of students. The evaluation does not only highlight the ability of the students through the final results of the tests, but it is also necessary to follow up the students mistake or error in responding to each problems by tracking the occurrence of the wrong response.

The facts on the field show that learning in university still tends to emphasize the skills of doing the questions, while the conceptualunderstanding is only given in a very short time so that students often make mistakes in solving problem. For example, for trigonometric material. According to Yulandari (2012) trigonometry is a material that is considered difficult by most students in mathematics subjects so that students experience confusion in its application. Furthermore, Aqillah (2012) states that trigonometry is a material that is considered difficult by students, especially on proving trigonometric identities that requires a precise understanding of concepts and high accuracy.

Based on the interviews to the lecturers of basic mathematics II, in that course, students will usually be difficult and have errors in proving trigonometric identity. If this situation continues to occur then most likely other errors will occur which ultimately leads to low student learning outcomes. This problem is certainly very necessary to overcome. One effort that can be done for example by analyzing the types of errors experienced by students so that lecturers can determine follow-up to overcome the problem.

Watson (in Asikin, 2002) categorizes 8 types of errors in working on the problem, as follows:

1) **Inappropriate data (id).**
   In this case the student tries to operate at the right level on a problem, but selects an inappropriate information or data.
2) **Inappropriate procedure (ip).**
   In this case the student tries to operate at the right level on a problem, but he uses the wrong procedure or the wrong way.
3) **Omitted data (od).**
   Students omitted one or more data in solving the problems. Thus the settlement is incorrect. Perhaps the student response did not find the right information, but the students were still trying to operate at the right procedure.
4) **Omitted conclusion (oc).**
   The symptom of the omitted conclusion is the student shows the reason at the right procedure then fails to conclude.
5) **Responselevel conflict (rlc).**
   Symptoms associated with omitted conclusion is response levels conflict. In this situation the student demonstrates an operating competition at a certain level and then lowers to lower operations, usually to conclusions.
6) **Indirect manipulation (im).**
   The reasons are not sequential but conclusions are obtained and generally all data is used. A correct answer is obtained by using simple reasons and illogical or random pouring. These

symptoms are observed as indirect manipulations.

7) Skill hierarchy problem (shp).

The expression of skills hierarchy problem is shown among others that students cannot solve the problem because of the lack or unseen skill ability.

8) In addition to the above seven categories (above other / ao).

Student errors that are not included in the above seven categories are grouped in this category. Errors that fall into this category include incorrect copying of data and not responding.

This research emphasizes that to analyze the students errors on trigonometric material especially in the proving trigonometric identities used Watson criteria by checking each step what is done by the student concerned.

II. METHOD

This study uses a qualitative approach that is used to explore and understand the meaning that by some individuals or groups of people considered as a social or humanitarian problem (Creswell, 2009: 17). The social problemsin this study referred to the student's mistakes in proving trigonometric identities. This type of research is descriptive. Descriptive research is a study that aims to create a systematic, factual, and accurate description of the facts and traits of the subject of research (Suryana, 2010: 18). The subjects of this study are 33 mathematics students strata 1 State University of Malang consisting of 2 high mathematical skilled students, 5 mediummathematical skilled students, and 26 lowmathematical skilled students. The grouping of these three categories refers to the scale of the assessment set by Arifin (2009), namely: (1) high mathematical ability if 80 ≤ score ≤100, (2) medium mathematics skill if 60 ≤ score <80, (3) low mathematics skill if 0 < score<60. The instrument used to collect data is test and interview. The test consisted of three proving trigonometrics identities problems that could be solved in various ways.

In this study, the criteria used to analyze students' errors are Watson's criteria which include: (1) improper data, (2) improper procedures, (3) missing/ omiteed data, (4) missing/ omitted conclusions, (5) responselevel conflicts, (6) indirect manipulation, (7) skill hierarchy problem, and (8) other categories.

III. RESULTS AND DISCUSSION

This research was conducted on October 10, 2017. A total of 33 students were asked to work on test questions about proving trigonometric identities consisting of 3 items. Of the 33 students, there was 1 student who successfully solve all problems correctly and 32 students who made mistakes consisting of 1 student with high mathematics skill, 5 students with medium mathematics skill, and 26 students with low mathematic skill. The test problems given to know the type of errors or mistakes made by students are as follows.

Prove the following identities trigonometric!

1. $\frac{\sec t + \tan t}{\cos t - \tan t - \sec t} = -\csc t$
2. $2\sin^2\theta\cos^2\theta + \cos^4\theta = 1 - \sin^4\theta$
3. $\tan 2x\,(\cos x + \cos 3x) = \sin x + \sin 3x$

From the results of the test, students are grouped into 3 categories, namely students with high, medium, and low mathematics skills. Furthermore from each group is determined by the students who make the most mistakes so that it can represent all the subjects. Then the analysis of student answer sheet to know the types of mistakes made by students with high, medium, and low mathematics skills.

The errors analysis of a highly mathematics skill subject (ST) in proving trigonometric identity on problem 2 can be seen in Fig 1.

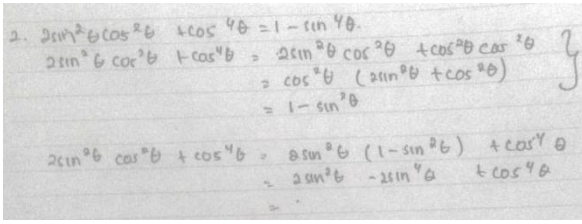

Fig 1. ST's answer for problem number 2

In Figure 1 it can be seen that ST can not solve problemnumber 2 because of lack of skill hierarchy so that he fails to prove trigonometric identity on problem 2.

The errors analysis of medium mathematics skill subject (SS) in proving trigonometric identities for problem number 1, 2, and 3 can be seen in Fig 2, Fig 3, and Fig 4.

In the Fig. 3, it can be seen that SS is careless in changing algebraic form, SS changed the form $1 - \sin^2 t - \sin t - 1$ to $\sin t\,(\sin t - 1)$, whereas it should be $\sin t\,(-\sin t - 1)$. This indicates that the SS experienced another category error that caused him to fail to prove the identity of trigonometry on problem number 1.Then at Fig 3, it can be seen that SS incorrectly changed the form $\sin^2\theta + 1$ to $\cos^2\theta$. In addition, SS is also wrong to change the form $\cos^2\theta$ to $1 - \sin^4\theta$. This indicates that the SS experienced an incorrect procedure error which resulted in the SS not being able to prove the identity of trigonometry on problem 2.While in Figure 3 it appears that the SS is unable to prove the identity of trigonometry because the SS has difficulty manipulating the form of trigonometric identities and the lack of skills to prove.

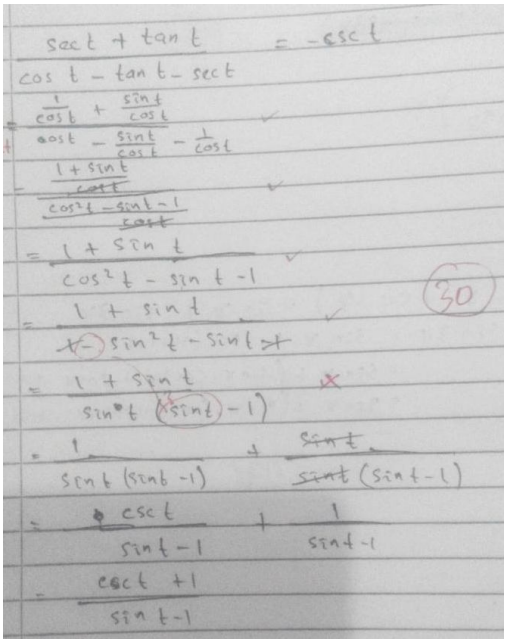

Fig.2. SS's answer for problem number 1

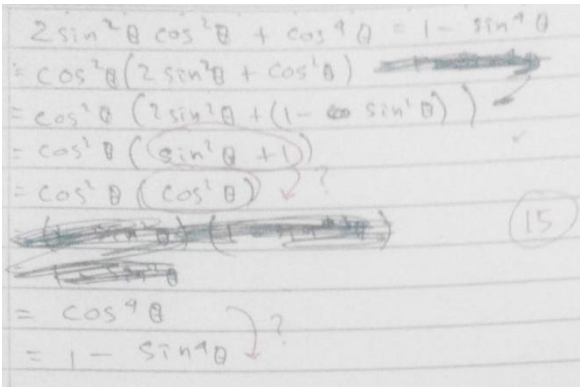

Fig.3. SS's answer for problem number 2

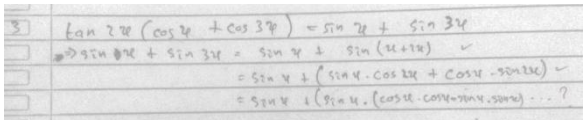

Fig.4. SS's answer for problem number 3

The errors analysis of low mathematics skill subject (SR) in proving trigonometric identities for problem number 1 and 3 can be seen in Fig. 5 and Fig.6.

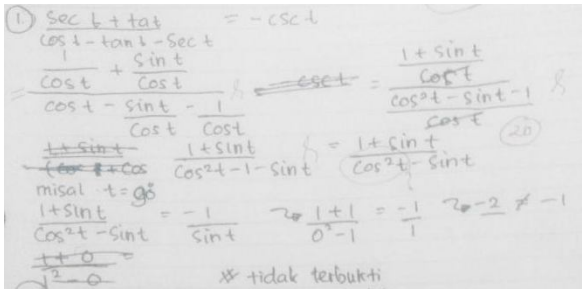

Fig. 5. SR's answer for problem number 1

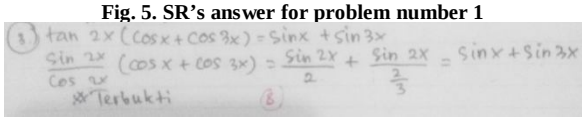

Fig. 6. SR's answer for problem number 3

In Figure 5 it appears that SR is experiencing an error because the data is missing or e omitted the data. He wrote the form $\cos^2\theta - 1 - \sin t$ to become $\cos^2\theta - \sin t$ .In addition, SR also experienced indirect manipulation error because SR uses the logical reason that is by taking the example t= 90^0 substituted on the final equation he found, whereas the equation is not correct. The omitted data error and also the indirect manipulation resulted in SR failing to prove the trigonometric equation in question number 1. Then in Figure 6 it shows that SR was wrong in changing the form of trigonometric identity. This indicates that the SR experienced an error using an improper procedure.

Based on the results of the research, it was found that the errors according to Watson's criterion by the subjectswith high, medium or low mathematics skill in solving the problem of proving trigonometric identity are missing/omited data, improper procedure, skill hierarchy problem, indirect manipulation and other categories. The missing/omitted data was done by a low-skilled subject, this is marked by the loss of one of the data written in the procedure that caused the substantiation of the subject to be inaccurate. This result is in line with the statement from Aqillah (2012) that learners make mistakes in writing data in proving trigonometric identity. Inappropriate procedure performed by high, medium and low-skill subjects. This inappropiate procedure was done by high, medium, and low skill subject in operating algebraic forms. This result is in accordance with a statement from Marsetyorini (2012) which states that the student's mistakes in working on algebraic questions lie in the way they operate algebraic forms. The error was caused by the subject that lack of understanding of the operating principle on algebra and square roots. This result is in line with Octaviano's (2012) assertion that the error in completing the algebraic form operation occurs due to student's lack understanding of the principle relating to algebraic operations. The skill hierarchy problem error is done by high, medium, and low-skill subjects. They unabled to manipulate trigonometric identities so that the proof becomes incomplete. This result is in line with Sari (2013) and Sahriah (2012) which states that the error in solving the math problem is not able to manipulate. In addition, Lado (2012) says that the student's mistake in manipulating the algebraic form is an error in manipulating the form of trigonometry.

Indirect manipulation errors are made by medium and low-skill subjects. The error of indirect manipulation is in the form of getting answers without reason or using the illogical way that the subject used to caused because the subject in a hurry in working on the problem in other words the subject is not careful. This result is in line with Sari (2012) statement that the students mistake in working on algebric problem is error because get answer without reason. Other

category errors are made by medium mthematics skill subjects. Another category is the inaccuracy of students in algebra to change the form so that student was wrong in interpreting the meaning of the problem. Agninditya (2014) states that the mistakes made by students in doing trigonometric problems caused by lack of accuracy.

Furthermore, based on the explanation of errors in solving the problem of trigonometric identity, it was found that errors according to the Watson criteria performed by high, medium or low-skill subjects were inappropriate procedures and skill hierarchy problems. Inappropiate procedurewas performed by high, medium, and low-skill subject. This inappropiate procedure is in the form of a student's mistake in changing a form of trigonometric identity into another form.

The skill hierarchy problem errors performed by high and low subjects. It is a mistake in changing the basic trigonometric identity so that trigonometric identities can not be proven. In accordance with the opinion of Aqillah (2012) that the mistakes made by learners in proving trigonometric identity is wrong in changing the existing trigonometrics identitywith another trigonometrics identity so that the step on proof is also wrong. This error is caused because the subject is unable to manipulate in proving trigonometrics identities.

CONCLUSIONS

Mathematics students error was studied and major conclusions are as follows:

1. The types of errors made by high mathematics skill subjects in proving trigonometric identities are improper procedures and skills hierarchy issues.
2. The types of errors made by medium mathematics skill subjects in proving trigonometric identities are missing data, inappropiate procedures, indirect manipulations, and skill hierarchy problem.
3. Types of mistakes made by low mathematics skill subjects in proving trigonometric identities are missing data, inappropiate procedures, indirect manipulation, skill hierarchy problem, and other categories.

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