COST-BENEFIT ANALYSIS OF TWO SIMILAR WARM STANDBY SATELLITE SYSTEMS SUBJECT TO FAILURE OF FBTP DUE TO: (A) GRIPPING AT ONE OF THE SEAL LOCATIONS AND SEIZURE OF ROTOR AND (B) RUPTURE OF TURBINE CASING CAUSED PROBABLY DUE TO EXCESSIVE PRESSURE RISE AND THERMAL STRESSES

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Abstract - In this paper we have taken failure of Fuel Booster Turbo Pump (FBTP) due to: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses. When the main operative unit fails then warm standby system becomes operative. Failure of FBTP due to gripping at one of the seal locations and seizure of rotor cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure of FBTP due to: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses. The repair is done on the basis of first fail first repaired.

Keyword - FBTP - Fuel Booster Turbo Pump, GSSMF- Gripping at one of the seal locations and seizure of rotor, RTCF- Rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses., MTSF – Mean time to system Failure.

INTRODUCTION

Chronology of ISRO's failed missions

The Friday failure of the GSLV is the third unsuccessful mission of the total seven of this indigenously developed space rocket.

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On April 15 this year, the third developmental flight of Geosynchronous Satellite Launch Vehicle (GSLV-D3) primarily for the flight testing of indigenously developed Cryogenic Upper Stage (CUS) could not accomplish the mission objectives.

Like the GSLV (carrying INSAT-4C on board) which failed on July 10, 2006, today's rocket was also fitted with the Russian cryogenic engine.

Unlike the April 15 mission when ISRO had faced anxious moments before the launch as it was the first time that the rocket was powered by the indigenous cryogenic engine, today's mission was considered "routine" and ISRO never expected trouble.

The Failure Analysis Committee comprising multi-disciplinary experts constituted by ISRO to look into the April 15 failure concluded that ignition of the CUS Main Engine and two Steering Engines have been confirmed as normal, as observed from the vehicle acceleration and different parameters of CUS measured during the flight.

Vehicle acceleration was comparable with that of earlier GSLV flights up to 2.2 seconds from start of CUS. However, the thrust build-up did not progress as expected due to non-availability of liquid hydrogen (LH2) supply to the thrust chamber of the Main Engine.

This failure is attributed to the anomalous stopping of Fuel Booster Turbo Pump (FBTP). The start-up of FBTP was normal. It reached a maximum speed of 34,800 rpm and continued to function as predicted after the start of CUS. However, the speed of FBTP started dipping after 0.9 seconds and it stopped within the next 0.6 seconds.

Two plausible scenarios have been identified for the failure of FBTP: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses.

The Failure Analysis Committee set up by ISRO after the July 10, 2006, unsuccessful mission, had concluded that the primary cause for the failure was the sudden loss of thrust in one out of the four liquid propellant strap-on stages (S4) immediately after lift-off at 0.2 sec. With only three strap-on stages working, there was significant reduction in the control capability.

However, the vehicle altitude could be controlled till about 50 secs. At the same time, the vehicle reached the transonic regime of flight and the vehicle altitude errors built up to large values, resulting in aerodynamic loads exceeding the design limits, thus leading to break-up of the vehicle.

Simulations and analyses of flight data and verification through calibration tests have led to the conclusion that the propellant regulator in the failed engine had much higher discharge coefficient in its closed condition.

The reason for this could be an inadvertent error in manufacturing, which escaped the subsequent inspection, and acceptance test procedures.

In brief:

- **December 25, 2010:** GSLV-F06 launch unsuccessful due to snag in stage-1
- **April 15, 2010:** GSLV-D3 developmental flight carrying GSAT4 onboard. Failure. Plunges into sea.
- **July 10, 2006:** Second operational flight of GSLV (GSLV-F02) with INSAT-4C onboard. Satellite could not be placed in orbit. Rocket falls into sea.
- **September 20, 1993:** First developmental launch of PSLV with IRS-1E on board. Satellite could not be placed in orbit.
- **July 13, 1988:** Second developmental launch of ASLV with SROSS-2 onboard. Satellite could not be placed in orbit.
- **March 24, 1987:** First developmental launch of ASLV with SROSS-1 satellite on board. Satellite could not be placed in orbit.
- **August 10, 1979:** First experimental launch of SLV-3 with Rohini Technology Payload onboard. Satellite could not be placed in the orbit.

In this paper we have taken failure of FBTP due to: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses. When the main operative unit fails then warm standby system becomes operative. Failure of FBTP due to gripping at one of the seal locations and seizure of rotor cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure of FBTP due to: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses. The repair is done on the basis of first fail first repaired.

Assumptions

1. λ_1, λ_2 λ_3 are constant failure rates due to warm standby, failure of FBTP due to: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$, $G_4(t)$.
2. The failure of FBTP due to gripping at one of the seal locations and seizure of rotor is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure of FBTP due to: (a) gripping at one of the seal locations and seizure of rotor and (b) rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts O, WS, GSSMF, RTCF,
Operative, Warm Standby, failure of FBTP due to gripping at one of the seal locations and seizure of rotor, rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses respectively

Subscripts ngssmf, gssmf, rtcf, ur, wr, uR
No failure of FBTP due to gripping at one of the seal locations and seizure of rotor, failure of FBTP due to gripping at one of the seal locations and seizure of rotor, failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses, under repair, waiting for repair, under repair continued from previous state respectively.

Up states – 0, 1, 2, 3, 10; Down states – 4, 5, 6, 7, 8,9,11, regeneration point – 0, 1, 2, 3, 8, 9, 10

States of the System

0(O_{ngssmf}, WS_{ngssmf}) One unit is operative and the other unit is warm standby and there is no failure of FBTP due to gripping at one of the seal locations and seizure of rotor of both the units.

1($GSSMF_{gssmf, urI}$, O_{ngssmf}) The operating unit failure of FBTP due to gripping at one of the seal locations and seizure of rotor is under repair immediately of Type- I and standby unit starts operating with no failure of FBTP due to gripping at one of the seal locations and seizure of rotor

2($RTCF_{rtcf, urII}$, O_{ngssmf}) The operative unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses and undergoes repair of Type II and the standby unit becomes operative with no failure of FBTP due to gripping at one of the seal locations and seizure of rotor

3($RTCF_{rtcf,\ urIII}$, O_{ngssmf}) The first unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses and under Type-III multispecialty repairman and the other unit is operative with no failure of FBTP due to gripping at one of the seal locations and seizure of rotor

4($GSSMF_{gssmf,uR1}$, $GSSMF_{gssmf,wrI}$) The unit failed due to GSSMF resulting from failure of FBTP due to gripping at one of the seal locations and seizure of rotor under repair of Type- I continued from state 1and the other unit failed due to GSSMF resulting from failure of FBTP due to gripping at one of the seal locations and seizure of rotor is waiting for repair of Type-I.

5($GSSMF_{gssmf,uR1}$, $RTCF_{rtcf,\ wrII}$) The unit failed due to GSSMF resulting from failure of FBTP due to gripping at one of the seal locations and seizure of rotor is under repair of Type- I continued from state 1and the other unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses is waiting for repair of Type- II.

6($RTCF_{rtcf,\ uRII}$, $GSSMF_{gssmf\ ,wrI}$) The operative unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses is under repair continues from state 2 of Type –II and the other unit failed due to GSSMF resulting from failure of FBTP due to gripping at one of the seal locations and seizure of rotor is waiting under repair of Type-I.

7($RTCF_{rtcf\ ,uRII}$, $GSSMF_{gssmf,wrII}$) The one unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses is continued to be under repair of Type II and the other unit failed due to GSSMF resulting from failure of FBTP due to gripping at one of the seal locations and seizure of rotor is waiting for repair of Type-II.

8($GSSMF_{gssmf,urIII}$, $RTCF_{rtcf,\ wrII}$) The one unit failure of FBTP due to gripping at one of the seal locations and seizure of rotor is under multispecialty repair of Type-III and the other unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses is waiting for repair of Type-II.

9($GSSMF_{gssmf,urIII}$, $RTCF_{rtcf,\ wrI}$) The one unit failure of FBTP due to gripping at one of the seal locations and seizure of rotor is under multispecialty repair of Type-III and the other unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses is waiting for repair of Type-I

10(O_{ngssmf} $RTCF_{rtcf,\ urIV}$) The one unit is operative with no failure of FBTP due to gripping at one of the seal locations and seizure of rotor and warm standby unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses and undergoes repair of type IV.

11(O_{ngssmf} $RTCF_{rtcf,\ uRIV}$) The one unit is operative with no failure of FBTP due to gripping at one of the seal locations and seizure of rotor and warm standby unit failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses and repair of type IV continues from state 10.

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$,
$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{10} = pG_1^*(\lambda_1) + q\,G_2^*(\lambda_2)$,
$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)}$, $p_{15} = q - q\,G_1^*(\lambda_2) = p_{12}^{(5)}$,
$p_{23} = pG_2^*(\lambda_1) + q\,G_2^*(\lambda_2)$, $p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}$,
$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)}$, $p_{30} = p_{82} = p_{91} = 1$,
$p_{0,10} = pG_4^*(\lambda_1) + q\,G_4^*(\lambda_2)$,
$p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$,
$p_{10,2} = q - q\,G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1$, $p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1$,
$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1$, $p_{30} = p_{82} = p_{91} = 1$
$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\ Ø_1(t) + Q_{02}(t)[s]\ Ø_2(t) + Q_{0,10}(t)[s]\ Ø_{10}(t)$
$Ø_1(t) = Q_{10}(t)[s]\ Ø_0(t) + Q_{14}(t) + Q_{15}(t)$
$Ø_2(t) = Q_{23}(t)[s]\ Ø_3(t) + Q_{26}(t) + Q_{27}(t)$, $Ø_3(t) = Q_{30}(t)[s]\ Ø_0(t)$,
$Ø_{10}(t) = Q_{10,0}(t)[s]\ Ø_{10}(t) + Q_{10,1}(t)[s]Ø_1(t) + Q_{10,2}(t)[s]\ Ø_2(t)$ (3-6)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (7)

where

$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^*\}\ [\ Q_{14}^*(s) + Q_{15}^*(s)\ ] + \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^*\}\ [\ Q_{26}^*(s) + Q_{27}^*(s)\ ]$
$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^*\}\ Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^*\}\ Q_{23}^*\ Q_{30}^* - Q_{0,10}^*\ Q_{10,0}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$MTSF = E[T] = \frac{d}{ds}\, ø_0^*(s) \Big|_{s=0}$$
$$= (D_1'(0) - N_1'(0)) / D_1(0)$$
$$= (\mu_0 + \mu_1 (p_{01} + p_{0,10}\ p_{10,1}) + (p_{02} + p_{0,10}\ p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\ p_{0,10} / (1 - (p_{01} + p_{0,10}\ p_{10,1})\ p_{10} - (p_{02} + p_{0,10}\ p_{10,2})\ p_{23}) - p_{0,10}\ p_{10,0}$$

where

$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$, $\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,

$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$, $\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t}$, $M_1(t) = p\, \bar{G}_1(t)\ e^{-\lambda_1 t}$

$M_2(t) = q\, \bar{G}_2(t)\, e^{-\lambda_2 t}$, $M_3(t) = \bar{G}_3(t)$,

$M_{10}(t) = \bar{G}_4(t)\, e^{-\lambda_3 t}$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\ A_8(t) + q_{29}^{(6)}(t)]\ [c]A_9(t)$

$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$,

$A_8(t) = q_{82}(t)[c]A_2(t)$, $A_9(t) = q_{91}(t)[c]A_1(t)$,

$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t)$

(8-15)

Taking Laplace Transform of eq. (8-15) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \tag{16}$$

where

$N_2(s) = \{\ \hat{q}_{0,10}\ \hat{M}_{10} + \hat{M}_0\ \}\ [\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7)}\ \hat{q}_{82}\ \} - \hat{q}_{12}^{(5)}\ \hat{q}_{29}^{(6)}\ \hat{q}_{91}\ ] + \{\ \hat{q}_{01} + \hat{q}_{0,10}\ \hat{q}_{10,1}^{(11)}\}[\ \hat{M}_1\{1 - \hat{q}_{28}^{(7)}\ \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\ \hat{q}_{23}\ \hat{M}_3 + \hat{M}_2] + \{\ \hat{q}_{02} + \hat{q}_{0,10}\ \hat{q}_{10,2}^{(11)}\}\ [\{\hat{q}_{23}\ \hat{M}_3\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\ \hat{q}_{91}\ \hat{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7)}\ \hat{q}_{82}\ \} - \hat{q}_{12}^{(5)}\ \hat{q}_{29}^{(6)}\ \hat{q}_{91} - \{\ \hat{q}_{01} + \hat{q}_{0,10}\ \hat{q}_{10,1}^{(11)}\ \}[\ \hat{q}_{10}\ \{1 - \hat{q}_{28}^{(7)}\ \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\ \hat{q}_{23}\ \hat{q}_{30}\ ] - \{\ \hat{q}_{02} + \hat{q}_{0,10}\ \hat{q}_{10,2}^{(11)}\}\{[\ \hat{q}_{23}\ \hat{q}_{30}\ \{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\ \hat{q}_{91}\ \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s) + s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \tag{17}$$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz \quad \text{So that}$$

$$\bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \tag{18}$$

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t) \quad \text{So that}$$

$$\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s) \tag{19}$$

Similarly, we can find out

1. The expected busy period of the server when there is failure of FBTP due to gripping at one of the seal locations and seizure of rotor, and failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses in (0, t] - R_0
2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0, t] - H_0
3. The expected number of visits by the multispecialty repairman Type-III or Type-IV for repairing the identical units in (0, t] - W_0, Y_0.

Benefit-Function

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure of FBTP due to gripping at one of the seal locations and seizure of rotor, and failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0, t] is

$$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$$

$$= K_1 A_0 - K_2 R_0 - K_3 H_0 - K_4 W_0 - K_5 Y_0$$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is busy under repairing,

K_3 - cost per visit by the repairman type- I or type-II for units repair,

K_4 - cost per visit by the multispecialty repairman Type- III for units repair,

K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to failure of FBTP due to gripping at one of the seal locations and seizure of rotor and failure of FBTP due to rupture of turbine casing caused probably due to excessive pressure rise and thermal stresses increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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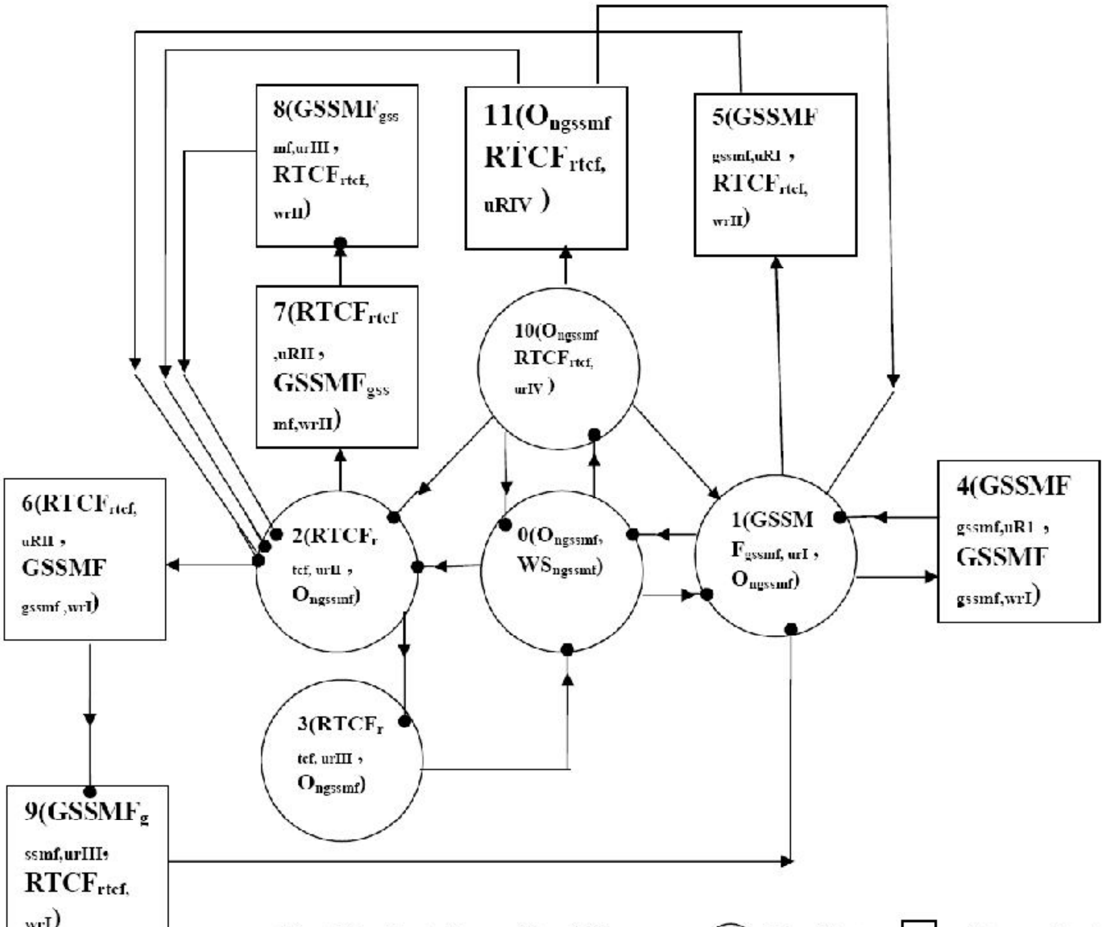

Fig. The State Transition Diagram ○ **Up-State** □ **Down-State**

●(GSSMF$_{gssmf,urIII}$, RTCF$_{rtcf, wrII}$) regeneration point

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