I. INTRODUCTION

Analyzing a signal in an adaptive manner finds application in various fields such as de-noising, compression, separation etc. In general a signal is expressed as a linear combination of basis functions. In Fourier or wavelet transforms basis functions are designed independently. For an adaptive technique the bases functions are constructed directly based on the information contained in the signal. A completely different approach to build an adaptive representation is the iterative method called “Empirical Mode Decomposition” (EMD) proposed by Huang et al. The main aim of this method is to detect the principal modes which represent the signal. The main advantage of this method is, it is able to separate stationary and non-stationary components from a signal. Mathematical modeling of EMD is difficult due to its non-linearity and lack of theory. A new approach to build adaptive wavelets capable of extracting AM-FM components of a signal was proposed by Jerome Gilles. The key idea is that such AM-FM components have a compact support Fourier spectrum. Separating the different modes is equivalent to segment the Fourier spectrum and apply some filtering corresponding to each detected support.

He showed that it is possible to adapt the wavelet formalism by considering distinct Fourier supports and then build a set of functions which form an ortho-normal basis. Based on this construction, he proposed an empirical wavelet transform (and its inverse) to analyze a signal. In case of EMD, the IMF’s obtained may or may not be orthogonal. Orthogonal bases are required for correct signal representation. A prior knowledge about the signal is required in case of EWT. As we are interested in developing adaptive representations, a better approach is proposed in this paper. The proposed technique decomposes the signals in an adaptive manner, such that all the components obtained are highly orthogonal. No prior knowledge about the signal is needed. Gram Schmidt orthogonalization principle is applied to the IMF’s generated by applying EMD to the signal. This will give information about the number of modes for applying EWT. Thus a signal can be decomposed into correct orthogonal components.

The remaining of the paper is organized as follows: In section II we recall some of the existing adaptive techniques used for signal analysis. In Section III, we describe the frame work of the proposed technique. Section IV shows experimental results on an ECG signal. Finally, we conclude and give some extensions in Section V.

II. EXISTING TECHNIQUES

A. Empirical Mode Decomposition

EMD aims to decompose a signal as a (finite) sum of N+1 Intrinsic Mode Functions (IMF).

\[
f(t) = \sum_{k=0}^{N} f_k(t)
\]

An IMF is an amplitude modulated-frequency modulated function which can be written in the form

\[
f_k(t) = F_k(t) \cos(\varphi_k(t))
\]

The IMF behaves as a harmonic component. Originally, the method of EMD to extract such IMF’s is a pure algorithmic method. Candidates for an IMF are extracted by first computing the upper and lower envelopes via a cubic spline interpolation from the maxima and minima of a signal. Then the mean envelope is computed. The first IMF is obtained by subtracting the mean envelope from the original signal. The remaining IMFs can be computed by...
repeating this algorithm on the successive residues. The interesting fact about this algorithm is that it is highly adaptable and is able to extract the non-stationary part of the original function. However, its main problem is that it is mathematically difficult to model. Consequently it is difficult to really understand what the EMD provides. For example, some problems appear when some noise is present in the signal.

B. Adaptive wavelet methods
Most known method is the wavelet packets in a basis pursuit framework. Even though the wavelet packets are useful in many applications, they use a constant prescribed ratio in the subdivision scheme, which limits their adaptability. Another approach, called the Malvar-Wilson wavelets, tries to build an adaptive representation by segmenting the Time domain signal to separate the time intervals containing different spectral information. While the original idea is interesting, it turns out that the temporal segmentation is a difficult task.

In, the authors propose a method, called the brushlets, which build an adaptive filter bank in the Fourier domain. It uses the idea of the Malvar-Wilson wavelets but segments the Fourier spectrum of the signal, instead of the signal itself. Conceptually the ideas in this work are really interesting; however the proposed construction is quite complicated and is also based on prescribed subdivisions. The last work we want to mention is a recent work of Daubechies called “synchrosqueezed wavelets”. This approach combines a classic wavelet analysis with a reallocation method of the time-frequency plane information. This algorithm permits to obtain a more accurate time-frequency representation and composition it is possible to extract specific modes. All the above methods use either a prescribed scale subdivision schemes or a smart utilization of the output of a classic wavelet analysis.

C. Empirical Wavelets
A method to build a family of wavelets adapted to the processed signal is proposed. In Fourier point of view, this construction is equivalent to building a set of band pass filters. One way to reach the adaptability is to consider that the filters’ supports depend on where the information in the spectrum of the analyzed signal is located. Here also consider a normalized Fourier axis which have a $2\pi$ periodicity, in order to respect the Shannon criteria, and restrict our discussion to $\omega \in [0, \pi]$.

Let start by assuming that the Fourier support $[0, \pi]$ is segmented into N contiguous segments. Let $\omega_k$ to be the limits between each segments. Each segment is denoted $\Lambda_n = [\omega_k, \omega_{k+1}]$. Centered around each $\omega_k$ we define a transition phase $T_n$ of width $\tau_n$. The empirical wavelets are defined as bandpass filters on each $\Lambda_n$. Thus define the empirical scaling function and the empirical wavelets by expressions of $(3)$ and $(4)$, respectively

\begin{equation}
q_0(\omega) = \begin{cases} 
1 & \text{if } |\omega - \omega_k| \leq \tau_n \\
\cos \left( \frac{1}{\tau_n} |\omega - \omega_k + \tau_n| \right) & \text{if } |\omega - \omega_k + \tau_n| \leq |\omega - \omega_k - \tau_n| \\
0 & \text{otherwise}
\end{cases}
\end{equation}

\begin{equation}
q_n(\omega) = \begin{cases} 
1 & \text{if } |\omega - \omega_k| \leq |\omega - \omega_{k+1}| - \tau_n \\
\cos \left( \frac{1}{\tau_n} |\omega - \omega_k + \tau_n| \right) & \text{if } |\omega - \omega_k + \tau_n| \leq |\omega - \omega_{k+1}| + \tau_n \\
\sin \left( \frac{1}{\tau_n} |\omega - \omega_k + \tau_n| \right) & \text{if } |\omega - \omega_k + \tau_n| \leq |\omega - \omega_k| + \tau_n \\
0 & \text{otherwise}
\end{cases}
\end{equation}

D. Empirical Wavelet Transform:
The Empirical Wavelet Transform is defined in the same way as for the classic wavelet transform. The detail coefficients are given by the inner products with the empirical wavelets:

\begin{equation}
W_n^f(n,t) = \langle f, q_n \rangle = \int f(t) q_n^* (t - t') \, dt'
\end{equation}

and the approximation coefficients by the inner product with the scaling function:

\begin{equation}
W_n^0(0,t) = \langle f, \phi_n \rangle = \int f(t) \phi_n^* (t - t') \, dt'
\end{equation}

The reconstruction is obtained by

\begin{equation}
f(t) = W_n^f(0,t) * \phi_n (1) + \sum_{n=1}^{N} W_n^f(n,t) * q_n (1)
\end{equation}

III. PROPOSED TECHNIQUE

The method involves applying EMD to the signal, the corresponding IMF’s are obtained. The obtained IMF’s are subjected to apply Gram-Schmidt orthogonalization procedure. The number of orthogonal components are estimated. The estimated components gives the number of modes for applying EWT. The EWT is applied to the signal to obtain the components.

A. Gram-Schmidt Orthogonalization Suppose we are given a signal set $\{S_1(t), S_2(t), \ldots, S_m(t)\}$
Find the orthogonal basis functions for this signal set \( \{s_k(t), \ldots, s_m(t)\} \). Where \( k \leq m \). First basis function is normalized version of the first signal. Successive basis functions are found by removing portions of signals that are correlated to previous basis functions and normalizing the result. This procedure is repeated until all basis functions are found.

**Fig. 3.** Processing flow of the proposed method

**IV. EXPERIMENTAL RESULTS**

We applied the proposed method on an ECG signal, because it is a signal with the properties—nonlinearity, nonstationarity. It is experienced that the described method give better decomposition than previous methods. A clear ECG signal is mixed up with baseline effects and random noise which is given as input.

**Fig. 6.** 3D plot of orthogonal components of ECG signal

**Fig. 7.** Input and reconstructed ECG signal

Adaptive Orthogonal Signal Decomposition Based on Empirical Mode Decomposition and Empirical Wavelet Transform
CONCLUSION

Proposed a simple and efficient adaptive technique to decompose the signals orthogonally. This method combines the Empirical Mode Decomposition and empirical wavelet transform. Gram Schmidt Orthogonalization (GSO) Procedure gives the proper orthogonal components. The method involves applying EMD to the signal, corresponding IMF’s are obtained. The obtained IMF’s are subjected to apply Gram-Schmidt orthogonalisation procedure. The number of orthogonal components are estimated. The estimated components gives the number of modes for applying EWT. Correct signal decomposition is done if all the bases are perfectly orthogonal. Experiments are carried out on an ECG signal.

ACKNOWLEDGMENT

The authors acknowledge Ms. Jayasree T C, Head of the Department, Electronics & Communication Engineering, KMCT College of Engineering, Calicut for her support for fulfilling this work and their valuable comments for improving this paper.

REFERENCES


Adaptive Orthogonal Signal Decomposition Based on Empirical Mode Decomposition and Empirical Wavelet Transform