

A TAGUCHI-BASED GREY RELATIONAL AND PRINCIPAL COMPONENT ANALYSIS FOR OPTIMAL DESIGN OF THRUST DENSITY AND TEMPERATURE OF TRAPEZOID IRONLESS LINEAR MOTOR

1WEN-JONG CHEN, 2CAI-XUAN LIN, 3JIA-RU LIN

1,2Department of Industrial Education and Technology, National Changhua University of Education, Taiwan, R.O.C.
E-mail: 1wjong@cc.ncue.edu.tw, 2asweet24123@gmail.com, 3apple790606@hotmail.com

Abstract- In this study, Taguchi based on the grey relational analysis (GRA) was integrated with principal component analysis for a trapezoidal ironless linear motor to determine the significant design parameters, including the magnet baseline, coil-winding width, coil-winding height and coil diameter, for the maximal thrust density and minimal temperature. Compared with the original motor, the thrust density and temperature were increased by 14.10% and 9.24%, respectively. The simulation results show that the proposed approach effectively improve the performance of ironless linear motors, and it could be used as a critical reference for designing the linear motors.

Index Terms- Grey relational analysis, Taguchi method, Trapezoidal ironless linear motor.

I. INTRODUCTION

The ironless linear synchronous motor in which the mover of the finite length is based on the coil winding, which is covered by an nonmagnetic material and slides in the U shaped stator. Although the thrust force of ironless linear motors is inferior to that of iron-core linear motors, the ironless linear motors do not produce hysteresis, cogging force, and edge effects. Thus, they exhibit high dynamic performance and stable thrust force compared with the iron-core linear motors [1].

The ironless linear motors have been used in many different industries. In general, the type of permanent magnets used in most linear motors are rectangular. However, the magnetic field distribution of rectangular magnets can generate distortion in the airgap between the stator and mover [2].

In addition, increasing the thrust density and thermal dissipation is crucial objectives for improving the performance of ironless-type linear motors. To achieve these goals, numerous previous studies have been conducted to optimize the design of ironless linear motors based on various objectives and the shapes and dimensions of permanent magnets. Sadegh and Isfahani [3] proposed a multiobjective design optimization approach for independently improving the thrust, thrust ripple, and consumed magnet volume. They study employed a genetic algorithm (GA) to identify the optimal design of a rectangular permanent magnet for an ironless linear motor. Tavanaet al. [4] used an analytical method and GA to optimize the dimensions of stair step-shaped magnetic pole that reduced the airgap flux density harmonics. Zhang et al. [5] proposed and discussed the design laws for the main dimensions of a motor to obtain an optimal sinusoidal magnetic field and stable thrust.

Sunand Zhou [6] proposed a design optimization for improving the magnetic flux density and reducing the magnet volume. A GA was employed to identify the optimal motor dimensions for attaining high thrust density, low thrust ripple, and economical magnet consumption. To improve the efficiency and performance of permanent magnets, Lee and Gweon [7] proposed a multi segmented trapezoidal (MST) magnet array to replace rectangular magnets of an identical dimension. That study concluded that linear motors featuring MST magnet arrays can generate more actuating force per volume. Zhang et al. [8]-[9] proposed a trapezoidal permanent magnet-based linear motor exhibiting a predominantly sinusoidal no-load magnetic field to reduce the thrust ripple. In addition, some studies have indicated that trapezoidal permanent magnets exhibit lower thrust ripple and higher thrust force compared with rectangular ones. However, there are very few studies about optimization of trapezoidal permanent magnet in ironless linear motor.

In this paper, the original design parameters considered were the dimensions of the topline, baseline, and height of trapezoidal permanent magnetic poles in the stator, as well as the airgap, coil-winding width, coil-winding height, and coil diameter of the mover. The Taguchi method was used with an L$_{27}(3^9)$ orthogonal array based on grey relational grades and principal component scores to identify the crucial parameters with multiple quality characteristics while avoiding multicollinearity problems. The grey relational grade of GRA was used to identify crucial design parameters that facilitate the following four objective factors: 1) maximal thrust density; 2) minimal temperature.
A Taguchi-Based Grey Relational and Principal Component Analysis For Optimal Design of Thrust Density and Temperature of Trapezoid Ironless Linear Motor

II. DESCRIPTION OF THE TRAPEZOIDAL IRONLESS PERMANENT MAGNET LINEAR MOTOR

A. Initial Modeling

Figure 1 presents the structure of a trapezoidal ironless permanent magnet linear motor comprising two parts: a mover with only a copper winding, and a stator with a U channel composed of an array of parallel trapezoidal alternate-pole permanent magnets on both sides of the U-channel base. In this study, the original design parameters were the magnet topline, magnet baseline, magnet height, airgap, coil-winding width, coil-winding height, and wire diameter. Table 1 lists the values of the geometrical parameters of the original motor.

![Figure 1: A topograph of the trapezoid ironless linear motor](image)

**Table 1. Original geometrical parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet topline</td>
<td>L</td>
<td>168mm</td>
<td>Stator length(x)</td>
<td>H</td>
<td>80mm</td>
</tr>
<tr>
<td>Magnet baseline</td>
<td>Wm</td>
<td>28mm</td>
<td>Stator length(y)</td>
<td>W</td>
<td>8mm</td>
</tr>
<tr>
<td>Magnet height</td>
<td>hm</td>
<td>5mm</td>
<td>Mover length</td>
<td>W</td>
<td>110mm</td>
</tr>
<tr>
<td>Airgap</td>
<td>δ</td>
<td>1mm</td>
<td>Mover length(2)</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>Coil width</td>
<td>wm</td>
<td>5.0mm</td>
<td>Mover length(y)</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>Coil height</td>
<td>nh</td>
<td>5.0mm</td>
<td>Winding pitch</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>Wire diameter</td>
<td>d_w</td>
<td>0.4mm</td>
<td>Mover height(y)</td>
<td>W</td>
<td>1.79</td>
</tr>
<tr>
<td>Number of phases</td>
<td>M</td>
<td>3</td>
<td>Temperature</td>
<td>T</td>
<td>90.5°C</td>
</tr>
<tr>
<td>Pole pitch</td>
<td>r</td>
<td>24.8mm</td>
<td>Current</td>
<td>A</td>
<td>22I</td>
</tr>
</tbody>
</table>

B. Thrust Force and Thrust Force Density

The thrust forces induced by the coil winding are determined based on the current values and strength of the magnetic field. Therefore, the thrust force and thrust force density formulation can be expressed as follows:

\[
D = \frac{F}{V} = \frac{1}{(6\pi)} \int_{x_1}^{x_2} J_x \times B_y H \, dx \, dy
\]

where \( B_y \) is magnetic field of flux density in airgap, \( S \) is coil-winding area within the magnetic field \( B \). The thrust density \( D \) is defined as the thrust force divided by motor volume \( V \). Therefore, once the thrust force is fixed, higher thrust force densities allow more space to be conserved.

III. DETERMINING THE OPTIMIZATION PARAMETERS

A. Taguchi-based grey relation analysis (GRA)

The Taguchi method is a method for controlling quality through experimental design to attain near optimal quality characteristics for a specific objective [10]. However, the traditional Taguchi method can only be performed on optimizing one quality characteristic. To handle multiple quality characteristics, this study incorporated grey relational analysis (GRA) and principal component analysis (PCA) into the Taguchi method to identify the crucial design parameters and determine the importance of each parameter. The GRA based on grey system theory introduced by Deng in 1982 is useful for dealing with multi-input, incomplete, discrete data and unsure information [11]. Through the grey relational analysis, the grey relational grade will be utilized to evaluate the multiple quality characteristics.

B. Data preprocessing

Because the range of values and type of units in data sequences can differ, the data used in this study were preprocessed by normalizing the mean and variance of each sequence. In addition, data must be normalized when the sequence scatter range is excessive, or when the direction of the target value differs between sequences. Let \( m \) denote the number experimental data items, and \( l \) represent the number of quality characteristics. The original experimental results can be expressed as the series \( y_1, ..., y_i, ..., y_m \).

\[
Y_i = \{ y_i(1), y_i(2), y_i(3), ..., y_i(l) \}, i=1, ..., m
\]

Where \( x_i \) represents the \( i^{th} \) experimental results and is called the comparative sequence in GRA. Let \( y_0(k) \) is the original reference sequence.

\[
Y_0 = \{ y_0(1), y_0(2), y_0(3), ..., y_0(l) \}
\]

The value of elements in a reference data sequence
indicates the optimal value of the corresponding quality characteristic. \( y_i \) and \( y_j \) both includes 1 elements, \( l=1,\ldots,k \) and \( y_i(l) \) and \( y_j(l) \) denote the numeric value of \( k^\text{th} \) element in the original reference sequence and the comparative sequence. If the original data are for a quality characteristic that corresponds to the “the-larger-the-better” assumption, the original data are preprocessed using the following “the-larger-the-better” algorithm:

\[
Y_i^*(k) = \frac{y_i(k)}{y_{\text{max}}(k)} \quad (4)
\]

If the original data correspond to the “the-smaller-the-better” assumption, they are preprocessed using the following “the-smaller-the-better” algorithm:

\[
Y_i^*(k) = \frac{y_{\text{min}}(k)}{y_i(k)} \quad (5)
\]

If the original data have a specific optimal target value \( y_{\text{opt}}(k) \), the quality characteristic corresponds to the “the-more-nominal-the-better” assumption, and the original data are preprocessed using the following “the-more-nominal-the-better” algorithm:

\[
Y_i^*(k) = \frac{\min\{y_i(k), y_{\text{opt}}(k)\}}{\max\{y_i(k), y_{\text{opt}}(k)\}} \quad (6)
\]

Where \( \max\{y_i(k) \) and \( \min\{y_i(k) \) are the maximum and minimum values respectively of the original sequence \( y_i(k) \). Comparable sequence \( Y_i^*(k) \) is the normalized sequence of original data.

C. Principal component Score

Because multicollinearity among quality characteristics can cause the solutions to fall into a local optimization trap, Principal component Score (PCA) was performed to transform the correlated data into uncorrelated linear combinations, or principal components. The uncorrelated property of each principal component (i.e., the principal component score) was incorporated into the GRA to determine the degree of influence of each design parameter. The total variation in the principal components (i.e., the eigenvalues) is equal to that in the original data. To calculate the principal component scores, the eigenvalue \( \lambda_k \) and corresponding eigenvector \( \beta_k \) must be determined from a correlation matrix formed using the formula for Pearson’s correlation coefficient to compute each value.

\[
Y_i(k) = \sum_{i=1}^{n} x_i^*(j) \beta_i, \quad i=0,1,\ldots, m; k=1,2,\ldots, n \quad (7)
\]

Where \( Y_i(k) \) is the principal component score of the \( k^\text{th} \) component in the \( i^\text{th} \) series, \( x_i^*(j) \) is the normalized value of the \( j^\text{th} \) sequence, and \( \beta_i \) is the \( j^\text{th} \) element of eigenvector \( \beta_k \). The larger the value of \( Y_i(k) \) is, the better the performance of the product/process is.

IV. CALCULATING THE GREY RELATIONAL COEFFICIENT AND GRADE

A grey relational grade is the mean of grey relational coefficients of a given variable. The grey relational coefficient of \( y_i(k) \) and \( y_j(k) \) is expressed as follows:

\[
r\left( y_i(k), y_j(k) \right) = \frac{\min_{j=1,2,\ldots, m} \left\{ | y_j^*(k) - y_i^*(k) | \right\} + \frac{1}{\zeta}}{\max_{j=1,2,\ldots, m} \left\{ | y_j^*(k) - y_i^*(k) | \right\} + \frac{1}{\zeta}} \quad (8)
\]

Where \( 0<\zeta \leq 1 \) is the relative difference of \( k^\text{th} \) element between sequence \( X \) and the comparative sequence \( Y \), \( \Delta_{y_{\text{max}}} = \max_{i=1,2,\ldots, m} | y_i(k) - y_{\text{opt}}(k) | \), \( \Delta_{y_{\text{min}}} = \min_{i=1,2,\ldots, m} | y_i(k) - y_{\text{opt}}(k) | \), \( \zeta \in [0,1] \) is the distinguishing coefficients. A grey relational grade is the average of the grey relational coefficients.

\[
r(\gamma_i, y_j) = \frac{1}{n} \sum_{k=1}^{n} r\left( y_i(k), y_j(k) \right) \quad (9)
\]

V. FLOWCHART OF OVERALL SCHEDULE

![Flow chart of the overall research](image)

VI. RESULTS AND DISCUSSIONS

A. Select the appropriate orthogonal array and conduct the experiments

The design parameters for the trapezoidal ironless linear motor that were selected as initial factors were the dimension of the baseline (A), topline (B), and height (C) of trapezoidal permanent magnetic poles in stator, as well as the airgap (D), coil-winding width (E), coil-winding height (F), coil diameter (G) of the mover, and operation current (H). Table 2 lists the three levels of the initial factors (i.e., Levels 1, 2, and 3).
A Taguchi-Based Grey Relational and Principal Component Analysis For Optimal Design of Thrust Density and Temperature of Trapezoid Ironless Linear Motor

Table 2. Experimental factors and their levels

<table>
<thead>
<tr>
<th>Levels</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>26</td>
<td>22</td>
<td>3.8</td>
<td>5.5</td>
<td>3</td>
<td>0.4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>22</td>
<td>4.8</td>
<td>5.5</td>
<td>5</td>
<td>0.4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>26</td>
<td>5</td>
<td>5.5</td>
<td>6</td>
<td>0.5</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

An L$_{27}$ ($3^8$) orthogonal array comprising seven columns and 27 rows was selected after considering the factors and levels. The eight factors represent the trapezoid ironless linear motor design parameters. Table 3 presents the sum of the ordinal values for each factor level from Taguchi-based GRA.

B. Find the principal component score

To compute the GRA with PCA, the distinguishing coefficient $\xi$, which was substituted for the grey relational coefficient in (7), was set at 0.5. The data preprocessing is that the results of each Taguchi experiment with various measurement units and ranges were appropriately normalized for transformation into dimensionless parameters with values ranging from zero to one. The correlational matrix, which describes the correlation among the quality characteristics, was formulated by applying Pearson’s correlation coefficient formula to calculate the principal component score of each experimental result for the four objective functions. Subsequently, the eigenvalues and eigenvectors were obtained. Thus, the scores of each principal component for all experimental results were computed using (8)-(9).

C. Perform the GRA

Table 5 indicates that A3, B3, C3, D1, E2, F3 and G2 exhibited the highest grey relational grade for the factors A, B, C, D, E, F, and G, respectively. Therefore, with a baseline of 26 mm (Level 1), topline of 24 mm (level 3), magnet height of 5 mm (Level 3), airgap of 0.8 mm (Level 1), coil-winding width of 5.5 mm (Level 2), coil-winding height of 5.5 mm (Level 3), and coil diameter of 0.4 mm (Level 2).

D. Compute the contribution rate and determine the most significant design factors

Analyze the experimental results by using the grey relational grade and ANOVA for selecting the levels of the most influent design factors. In Table 4, the larger the F value is, the more significant the effect of this parameter is. $F_{0.01, 3, 27}$ was 2.96 at the 95% confidence level. As indicated in the table, the $F$-values of factors A, E, F, and G were all greater than $F_{0.01, 3, 27}$, and the most significant effect was coil diameter G. The baseline A, coil-winding width E, coil-winding width F, and coil diameter G were the most significant parameters of the maximal thrust density and minimal temperature.

Figures 3(a)-(b) and 3(c)-(d) show the temperature distributions and magnetic flux density of initial model and Taguchi-based GRA. The greater thrust density and the lower temperature distribution for the results of Taguchi-based GRA are presented, compared with the initial model. With compared with the initial model, the thrust density has the increment of 9.24%, and the temperature distribution has the decrements of 14.10 % for Taguchi-based GRA, as shown in Table 5. The results show that the proposed method in this paper can really improve the operation performance of trapezoid ironless linear motors.
Fig.3. The temperature distribution (°C) and magnetic flux density (T) (a)-(b) Initial model, (c)-(d) Taguchi-based GRA

Table 5. The comparison of initial model and Taguchi-based GRA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
<th>x₄</th>
<th>Thrust density (N/10⁻⁶mm²)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial model</td>
<td>28.00</td>
<td>5.00</td>
<td>5.00</td>
<td>0.40</td>
<td>0.482511</td>
<td>122.57</td>
</tr>
<tr>
<td>Taguchi-based GRA</td>
<td>30.00</td>
<td>5.50</td>
<td>5.50</td>
<td>0.40</td>
<td>0.523634</td>
<td>105.29</td>
</tr>
</tbody>
</table>

CONCLUSIONS

In this study, a optimization method combined Taguchi-based GRA with PCA was employed to efficiently investigate the effects of magnet topline, magnet baseline, airgap length, winding coil height, and coil diameter on the maximal thrust density, and minimal temperature. The first stage involved using the Taguchi method to determine the crucial design parameters. The second stage involved using the grey relational analysis to determine the appropriate values of the crucial design parameters. The research results indicated that the designers or developers can apply the proposed approach to enhance the design efficiency of trapezoid ironless linear motors or extend the approach to other industrial applications.

REFERENCES