POSITION CONTROL AND ANTI-SWING CONTROL OF OVERHEAD CRANE USING OPTIMAL CONTROL

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Abstract- A crane is a machine equipped with a hoist, wire ropes or chains which is mainly used both to lift and lower materials and move them horizontally to different places. However the underactuated structure of the crane makes difficult to control. The most important control requirement is to make the trolley position converge to desired position accurately with less payload swing which otherwise hinders the precise positioning of the load. In the practical situations, however, the achievement of both transporting trolley to target position and reducing payload sway is not always easy. The main objective is to employ a linear quadratic regulator for controlling the trolley position and swing motion which give better performance than simple PD and PID control. Moreover, the proposed method is then provided with disturbances and then compensated with linear quadratic Gaussian controller.

I. INTRODUCTION

The main purpose of controlling an overhead crane is transporting the load as fast as possible without causing any excessive swing from one position to another. However, most of the common overhead crane results in a swing movement when payload is suddenly stopped after a fast motion [1]. The overhead crane are used mainly in the industries, hazardous environments like nuclear power plant due to its low cost, easy assembly, precise positioning of load and less maintenance. The swing motion can be reduced by adjusting the speed of the motor but it will be time-consuming process. Moreover, a crane needs a skillful operator to control both the position and swing manually based on experiences to stop the swing immediately at the desired position. The inefficiency of controlling the crane also might cause accident and create harm to the people and the surroundings.

Various attempts in controlling cranes system based on open loop system were proposed. Earlier open loop time optimal strategies were applied to the overhead crane by many researchers such as discussed in [2],[3]. They came out with poor results because open loop strategy is sensitive to the system parameters (e.g. rope length) and could not compensate for wind disturbances. Another importance of open loop strategy is the input shaping introduced by Karnopp [4], Teo [5] and Singhose [6]. However the input shaping method is still an open-loop approach. Hubbel et al. [7] used an open-loop method to control the motion of the crane. In this open loop control method, the input control profile was determined in such a way that unwanted oscillations and residual pendulations were eliminated. However their approach was applicable, but the open loop control scheme is not robust to disturbances and parameter uncertainties [8].

Moreover, a feedback PID anti-swing controller is developed in [9] to control of an overhead crane. Ahmad et al. [10] used a hybrid input-shaping method to control of the crane. Wahyudi and Jalani [11] introduced fuzzy logic feedback controller to control the intelligent crane. They also presented an optimal control method is used in [12] to control the dynamic motion of the crane. Here, minimum energy of the system and also integrated absolute error of payload angle are assumed as their optimization criterion. Zhao and Gao [13] studied the control of the overhead crane. They proposed a fuzzy control method to control the input delay and actuator saturation of the system. Nazemizadeh et al. [14] studied tracking control of the crane. Furthermore, Nazemizadeh [15] presented a PID tuning method for tracking control of a crane.

In this paper, overhead crane is provided with a Linear Quadratic Regulator to control both swing and position of the crane and the results are compared with PD and PID control. It is then provided with a Linear Quadratic Gaussian to nullify the external disturbance. The structure of this paper is as follows. The modelling of the crane is explained in section 2. Section 3 highlights the development of model-based soft sensor. Section 4 discusses about the crane with conventional PD and PID controller. Section 5 discusses about the crane with Linear Quadratic Regulator. Section 6 discusses about the crane with Linear Quadratic Gaussian. Simulation results is given in section 7. Conclusion and future scope is discussed in section 8 and 9 respectively.

II. MODELLING OF CRANE

Fig. 1 shows a schematic diagram of the crane considered in this paper. Due to the fact that only
planar motion of crane is considered in this paper, there are two independent coordinates namely \( y \) and \( \theta_y \) to describe the trolley position and the swing angle of the payload respectively.

Since the mass of the rope is small enough as compared to the payload mass \( m_p \), it is considered as massless. The non-linear dynamic model of overhead crane prototype is derived using Lagrange equations.

\[
(m_t + m_p)\ddot{y} + m_p g y \sin \theta_y = F_y \\
\cos \theta_y \ddot{\theta}_y - g \sin \theta_y = \tau_d \cos \theta_y
\]  

(2)

By assuming small motion of \( \theta_y \), the following linearized model of the crane is obtained.

\[
(m_t + m_p)\ddot{y} + m_p g y \ddot{\theta}_y = F_y \\
y + \dot{y} + g \theta_y = 0
\]  

(3)

(4)

In the above equation, \( m_t \) is the mass of the trolley, \( m_p \) is mass of the payload, \( l \) is the length of the rope, \( y \) is the position of the trolley, \( \theta_y \) is the swing angle and \( F_y \) is the force provided by the dc motor. Thus the state space model of the overhead crane can be obtained as

\[
\dot{X} = AX + Bu
\]  

(5)

\[
Y = Cx + Du
\]  

(6)

where

\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & m_p g & 0 \\ 0 & 0 & \frac{m_p g}{m_t} \frac{L}{l} & 0 \\ 0 & 0 & \frac{m_p g}{m_t} \frac{L}{l} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = [0]
\]


\[
X = [y, \dot{y}, \theta_y, \dot{\theta}_y]^T
\]

The translational motion of the trolley is driven by DC motor. Therefore, to obtain the entire model of the crane, the motor dynamic is modeled according to equivalent DC motor circuit. The equivalent circuit of DC motor has armature resistance \( R \), inductance \( L \), motor inertia \( J \), torque constant \( k_T \), input voltage to the dc motor \( V \), armature current \( I \) and damping coefficient \( B \). The rotational motion is converted to translational motion through the mechanical part (pulley or gear) with radii of \( r \). The dynamics of dc motor circuit is given by the following equations

\[
V = R I + L \frac{dI}{dt} + k_T \theta
\]  

(7)

\[
T = k_T \theta
\]  

(8)

\[
J \ddot{\theta} + B \dot{\theta} = T
\]  

(9)

III. DEVELOPMENT OF MODEL-BASED SENSOR

Most of the overhead crane use two controllers for controlling both trolley position and swing of the crane payload. Therefore two sensors are needed to measure the trolley position \( Y(s) \) and swing angle \( \theta_y(s) \). The latter is usually installed on the load side. Since the use of sensors on the load side is troublesome, a model-based soft sensor is proposed to provide output estimation of the plant. The dynamic information from trolley position \( Y(s) \) is given to the developed soft sensor. It produces output estimation of the payload motion that will be used for feedback signal to the controller.

A model-based soft sensor is adopted in the proposed method. The proposed model based soft sensor is used to estimate the swing angle of the payload based on the position of the trolley. Consequently, dynamic model is used as model-based soft sensor. According to the equations, the swing angle of payload is estimated by using the following:

\[
\frac{\theta_y(s)}{Y(s)} = \frac{\frac{s^2}{g^2} + \frac{L}{2}}{s^2 + \frac{B}{J} + \frac{L^2}{2}}
\]  

(10)

where \( \theta_y(s) \) and \( Y(s) \) are estimated swing motion of the payload and trolley motion in Laplace domain respectively. It is shown that the proposed model-based soft sensor is easily and practically implemented since it has a simple structure and depends only on the length of the rope \( l \) and acceleration due to gravity \( g \) which are easily known.

IV. CRANE WITH CONVENTIONAL PID AND PD CONTROLLER

Well known classical PID controllers are designed and used to evaluate the effectiveness of the proposed model-based soft sensor. The function of the controller is to control the payload position \( Y(s) \) so that it moves
to the desired position $y_d(t)$ as fast as possible without excessive swing angle $y_s(t)$. A PID controller is adopted to control the trolley position, while a PD controller is used for swing control. The controller gains for PID and PD are designed and optimized with simulation model by using Simulink response optimization library block. It is mainly a numerical time domain optimizer developed under MATLAB/Simulink environment. Hence the response obtained by the simulink optimization library block assists in time-domain-based control design by setting the required value of overshoot, settling time and steady state error.

### Table 1: Crane and motor parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_T$</td>
<td>0.23</td>
<td>Kg</td>
</tr>
<tr>
<td>$m_T$</td>
<td>1.073</td>
<td>Kg</td>
</tr>
<tr>
<td>$l$</td>
<td>0.3302</td>
<td>m</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
<tr>
<td>$r$</td>
<td>0.006</td>
<td>m</td>
</tr>
<tr>
<td>$R$</td>
<td>2.6</td>
<td>Ω</td>
</tr>
<tr>
<td>$L$</td>
<td>2.5 x 10^-3</td>
<td>H</td>
</tr>
<tr>
<td>$J$</td>
<td>2 x 10^-3</td>
<td>Kg/m²</td>
</tr>
<tr>
<td>$B$</td>
<td>5 x 10^-5</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$k_b$</td>
<td>0.00767</td>
<td>Nm/A</td>
</tr>
<tr>
<td>$k_t$</td>
<td>0.00767</td>
<td>V/s/rad</td>
</tr>
</tbody>
</table>

In order to realize fast motion with low value for overshoot, the PID controller is optimized. Moreover, in order to suppress the swing angle quickly, the PD controller is optimized. Thus there are five parameters to be optimized in order to have satisfactory control performance. The parameters, $K_p$, $K_i$, $K_d$, $K_p$, and $K_d$ which are the proportional, integral, and derivative gains for the position control and proportional, derivative gains for the anti-swing control. The optimization to obtain PID+PD gains for position and anti-swing crane control was done using Ziegler Nichols tuning method. Based on the result, the gains of $K_p$, $K_i$, $K_d$, $K_p$, and $K_d$ are shown in Table 2.

### Table 2: Optimized PID+PD position and anti-swing gains

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$K_p$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140.4</td>
<td>0.7</td>
<td>136.5</td>
<td>43.5</td>
<td>12.8</td>
</tr>
</tbody>
</table>

V. LINEAR QUADRATIC REGULATOR (LQR)

The stability is a major problem in an overhead crane system. If the state variables are known, then they can be utilized to design a feedback controller so that the input becomes $u = -Kx$. It is necessary to measure and utilize the state variables of the system in order to control the position and swing angle of the crane. The feedback control system is provided with a control signal $u(t)$ which is obtained as:

$$u(t) = -(K_{p_1} + K_{p_2} + K_{p_3} + K_{p_4})$$

Then the equation becomes:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \left(\frac{1}{m_2} + \frac{k_b}{m_2}ight)x_2 + \frac{m_1}{m_2}x_3 + \frac{m_1}{m_2}x_4 \\
\dot{x}_3 &= \frac{k_b}{m_2}x_2 + \frac{m_1}{m_2}x_3 + \frac{m_1}{m_2}x_4 \\
\dot{x}_4 &= \frac{k_b}{m_2}x_2 + \frac{m_1}{m_2}x_3 + \frac{m_1}{m_2}x_4
\end{align*}$$

Arranging in matrix form, it is expressed as:

$$\dot{X} = Ax + Bu$$

$$Y = Cx + Du$$

where

$$A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad C = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad D = \begin{bmatrix}
0
\end{bmatrix}$$

Which is of the form $\dot{x} = Ax - kx = (A - BK)x = Hx$ (15)

To minimize the performance index $J$, consider the following two equations,

$$J = \int_0^\infty X^T(t)X(t)dt = X^T(0)PX(0)$$

$$HP + PH = -I$$

This compensated system is considered to be an optimal system which results in a minimum value for the performance index. The main advantage of using the quadratic optimal control scheme is that the system designed will be stable, except in the case where the system is not controllable. The matrix ‘$P$’ is determined from the solution of the matrix Riccati equation. This optimal control is called the Linear Quadratic Regulator (LQR). The optimal feedback gain matrix $K$ can be obtained by solving the following Riccati equation for a positive-definite matrix ‘$P$’.

$$A^TP + PA - PBR^{-1}B^TP + Q = 0$$

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Let $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ \hspace{1cm} (19)

$R = \begin{bmatrix} 0.01 & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$ \hspace{1cm} (20)

Tuning of Q and R matrix, P matrix is determined. So the optimal feedback gain is obtained as:

$$k = R^{-1}B^TP$$ \hspace{1cm} (21)

$$u = -10x_1 - 13.4966x_2 + 31.2263x_3 + 7.8037x_4$$ \hspace{1cm} (22)

This control signal yields an optimal result for any initial state under the given performance index. In LQR the cost function which is to be minimized is given by

$$J = \int_0^\infty (x^TQx + u^TRu) \, dt$$ \hspace{1cm} (23)

The two matrices $Q$ and $R$ are selected by design engineer by using Bryson rule. Selecting $Q$ large means that, to keep J small. On the other hand choosing $R$ large means that the control input $u$ must be smaller to keep $J$ small One should select $Q$ to be positive semi definite and $R$ to be positive definite. This means that the scalar quantity $k^TRQk$ is always positive or zero at each time $t$. The best value of the $Q$ & $R$ matrix is calculated by checking the step response of the system (with LQR).

VI. LINEAR QUADRATIC GAUSSIAN (LQG)

The linear-quadratic-Gaussian (LQG) control problem is one of the most fundamental optimal control problems. It mainly deals with uncertain linear systems affected by additive white Gaussian noise, having incomplete state information (i.e. not all the state variables are measured and available for feedback) and undergoing control subject to quadratic costs. Also the solution is considered unique and constitutes a linear dynamic feedback law that is easily computed and implemented. The LQG controller is simply the combination of a Kalman filter i.e. a linear-quadratic estimator (LQE) with a linear-quadratic regulator (LQR). The separation principle guarantees that these can be designed and computed separately. LQG control applied to both linear time invariant systems as well as linear time-varying systems. LQR provides optimal state feedback which minimizes a quadratic cost about the states. Kalman filter provides optimal state estimation. In stochastic control, the above two give the Linear Quadratic Gaussian (LQG) controller. Thus the state space model of the overhead crane can be obtained as

$$\dot{\hat{x}} = Ax + Bu + \xi(t)$$ \hspace{1cm} (24)

$$y = Cx + Du + \phi(t)$$ \hspace{1cm} (25)

In LQG design the disturbances are added in the system mainly the process and measurement noise respectively which are assumed to be zero mean with power spectral density matrices of $W$ & $V$. The Kalman Gain can be found out by using the equation

$$K_t = VL_tV_t^{-1}$$ \hspace{1cm} (26)

Where $Y = VL_t \geq 0$ is the unique positive-semidefinite solution of the algebraic Riccati equation

$$YA_t + A_tY - YC_tV_t^{-1}Y + W = 0$$ \hspace{1cm} (27)

First, calculate $\hat{x}(t)$ estimate the full state $x(t)$ using the available information. Secondly, apply the LQR controller, using the estimation $\hat{x}(t)$ in place of the true (now unknown) state $x(t)$.

$$W = \begin{bmatrix} U & N_x \\ N_x^T & R \end{bmatrix} \text{ and } V = \begin{bmatrix} \xi & N_{\phi} \\ N_{\phi}^T & \phi \end{bmatrix}$$ \hspace{1cm} (28)

where $Q$ and $R$ are weighing matrix of LQR and $\xi$ and $\phi$ are the covariance of plant noise $\xi(t)$ and measurement noise $\phi(t)$ and $N_x$ & $N_{\phi}$ are zero matrices.

VII. SIMULATION RESULTS

MATLAB software package is used to determine the response of the system. The Simulink model of the system with optimized values of PD and PID controller is created in MATLAB. Tuning of the Q & R matrix is done by separate coding. The regulation of position and swing angle is determined with tuning of Q & R matrix and the tracking of the crane also determined. The Simulink model of the system is developed. The figure 2 shows the position and swing angle control using optimized values for PD and PID controller. From figure 3, figure 4 and figure 5, the system is regulated at 2 sec, 3 sec and 5 sec respectively and the system consists of small number of overshoot and undershoot when compared to PD and PID controller. The system is precisely regulated at this condition. The figure 6 shows position control using LQG. LQG control the position without undershoot in spite of the disturbances.
CONCLUSION

In this paper, an LQR system was designed for the crane which results in a minimum value for the performance index. Also, the control law given by equation (22) yields optimal result for any initial state under the given performance index. Both the transient and steady state response of the system is improved with LQR controller. This design based on the quadratic performance index yields a stable control system for the overhead crane. The system is then provided with external disturbances. LQG controls the position without undershoot in spite of the disturbances.

FUTURE SCOPE

A Linear Quadratic Gaussian controller which also eliminates swing due to the disturbance affect into the system is not considered in this paper. It can be done using phase lag compensator.

REFERENCES