SYSTEM RELIABILITY MAXIMIZATION FOR STOCHASTIC-FLOW NETWORK SUBJECT TO TOTAL LEAD-TIME BASED ON RANDOM WEIGHTED GENETIC ALGORITHM

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Abstract - System reliability is an important performance index for many real-life systems, such as logistic information systems, electric power systems, computer systems, and transportation systems. These systems can be modeled as stochastic-flow networks (SFNs) composed of arcs and nodes. In this paper, we investigate components assignment problem for stochastic flow networks subject to two constraints namely total lead-time, and system reliability. A new approach based on random weighted genetic algorithm (RWGA) is proposed for searching an optimal components assignment which leads to maximizing system reliability and minimizing total lead time. The results revealed that an optimal components assignment leads to the maximum reliability and minimum total lead-time using the proposed approach.

Keyword - Component Assignment Problem, Genetic Algorithm, Lead-Time, System Reliability.

I. INTRODUCTION

The reliability of a Stochastic Flow Network (SFN) under a time constraint is defined as the probability that the SFN can send the required amount of data from the source to the sink within a specified amount of time [6].

The component assignment problem (CAP) aims to find the optimal arrangement of n available components to m positions in a system to maximize the system reliability. Components assignment problem (CAP) for stochastic-flow network has widely been accepted by the manufacturers and researchers. However, most of the previous research focused on the CAP subject to one constraint such as assignment budget, or lead-time but not focused on two constraints.

Components assignment problem for stochastic-flow network (SFN) under assignment cost has been solved by using genetic algorithm (GA), Lin and Yeh [1]. Furthermore, Harper el al. [22] used GA to find an optimal assignment of projects to students. Next, PC.Chu and J.F Beasley [23] have presented a GA-based heuristic for solving the generalized assignment problem to find the minimum cost assignment of n jobs to m agents. As a special case of a generalized assignment problem, Yen-Zen wang [24] has used GA methods for teacher assignment problems and the obtained results showed that the proposed technique can save a significant time spent on teacher assignments problems. Considering each link has a lead-time and capacity, the transmission time of network paths can be evaluated and determined the quickest one [4-7]. Furthermore, the idea of the quickest path has been extended to evaluate the reliability of a SFN, [6], [9], and [10]. Reference [11], discussed each component possessing a lead-time, and proposed a genetic algorithm to search the set of components that maximize the system reliability such that the total lead-time cannot exceed a specified amount.

This paper discusses components assignment problem for the case where each component has two attributes: a lead-time and system reliability. The main goal is to find the optimal components that maximize system reliability and minimize the total lead-time. Furthermore, a new algorithm based on random weighted genetic algorithm (RWGA) is proposed to solve the presented problem.

The rest of this paper is organized as follows: Section 2 deals with the needed notations, and section 3 presents the problem formulation. Next, Section 4 discusses the reliability evaluation under a time constraint. Section 5 explains the proposed algorithm based on RWGA. To demonstrate the effectiveness of the proposed algorithm, two practical examples included in Section 6 in order to demonstrate the proposed approach. A brief discussion about the obtained results is given in section 7. Finally, Section 8 draws conclusions and future work.

II. NOTATIONS

n Set of nodes.

m \{a_i| \leq e \leq m\} : set of arcs.

MPs Minimal paths.

np Number of minimal paths.

cmp Minimal path no. j; j = 1, 2, ..., np.

cn The number of available components.

cpk The components number k; k=1,2,...,cn.

cpk Leadtime of components cpk.

Lj The lead-time of mpj.

c(pk) The cost of the component cpk.

Rsd The system reliability to the demand d under time limit T.
X Capacity vector defined as \( X = (x_1, x_2, \ldots, x_e) \).

\( Ps \) Population size.

\( Maxgen \) Maximum number of generations.

\( gn \) Generation number.

\( pm \) GA mutation rate.

\( pc \) GA crossover rate.

### III. PROBLEM FORMULATION

Let \( CP = \{ cpk | 1 \leq k \leq cn \} \) be a set of available components, and \( B = \{ b_1, b_2, \ldots, b_m \} \) be the components assignment in which \( cpk \) is assigned to the arc \( ae \) if \( be = k \). The total lead-time with a specified components assignment \( B \) is given by \( S_l(B) = \sum_{e=1}^{m} l(b_e) \). Therefore, the mathematical programming formulation of the problem under study is given by:

**Maximize** \( R_{dT}(B) \)  \hspace{2cm} (1)

**Maximize** \( S_l(B) \)  \hspace{2cm} (2)

Subject to

\( b_e = k \), \( k \in \{1, 2, \ldots, cn\} \) for \( e = 1, 2, \ldots, m \).  \hspace{2cm} (3)

\( b_e \neq b_f \) for \( e \neq f \).  \hspace{2cm} (4)

\( L_j = \sum_{e=1}^{m} l(b_e) \) be the total lead-time of a MP, \( L_j \) is less than the time limit (T)  \hspace{2cm} (5)

### IV. RELIABILITY EVALUATION UNDER LEAD-TIME CONSTRAINT

Each component \( cpk \) has a maximum capacity \( M^k \), and capacity values of \( cpk \) vary from 0 – \( M^k \). In addition, the lead-time of component \( cpk \) is \( l(cp_k) \) and the system reliability of the candidate chromosome can be evaluated as follows:

**Step 1.** Check if the candidate chromosome satisfies constraint (8).

**Step 2.** Use the procedure described by Lin [1] to generate the capacity vector \( X^t \) corresponding to the path \( mp_j \).

**Step 3.** Calculate the network reliability of the chromosome:

\[ R_{dT} = \text{Pr}\{\bigcup_{i=1}^{n} \{ X | X \geq X^t \} \} \]

using the inclusion-exclusion rule given by Xue [15].

### V. PROPOSED APPROACH BASED ON RWGA

In this section, we develop an approach to solve the multi-objective optimization problem based on RWGA, which is used to determine the highest ranking solution to the problem. The initial inputs include data related to the components such as the lead-time, assignment cost, probability, the parameters of RWGA, and the network topology.

### VI. CROSSOVER, MUTATION AND SELECTION OPERATIONS

We use the modified uniform crossover and mutation presented in Hassan [9] to generate new offspring. The crossover operation is described as follows: Given the parents \( B_1 = (5, 2, 1, 3, 4, 6) \) and \( B_2 = (3, 1, 4, 2, 6, 5) \). Therefore, the new offspring is filled randomly by selecting genes from \( B_1 = (5, 2, 1, 3, 4, 6) \) and \( B_2 = (3, 1, 4, 2, 6, 5) \), which will be \( (5, 1, 1, 2, 4, 5) \). Where, the second and the third genes are equal, but the two different genes (3 and 6) are left free. The free genes (3 and 6) can be used to replace the second and the sixth genes in the offspring, and the offspring will be \( (5, 3, 1, 2, 4, 6) \). This crossover process is shown in Fig 1.

We note that swap mutation is used to avoid duplicated genes in a chromosome. Next, letting the parent \( B_1 = (5, 3, 1, 2, 4, 6) \), and the first and fourth genes are selected to swap their values. Then, the new offspring will be \( B_1 = (2, 3, 1, 5, 4, 6) \). The mutation process is shown in Fig 2.

### VII. DETERMINE THE FITNESS

Let \( Rd(T,i) \) and \( Sl(i) \) be the corresponding values for the solution \( i, i=1,2, \ldots, ps \).

**Step 1.** Find the normalized values of \( Rd(T) \) and \( Sl \) as follows:

**Step 1.1.** Normalized value for \( Rd(T) \):

\[ NRd(T(i)) = \frac{Rd(T(i))}{\max(Rd(T(1), Rd(T(2), \ldots, Rd(T(ps)))} \]

**Step 1.2.** Normalized value for \( Sl(i) \):

\[ NSl(i) = \frac{Sl(i)}{\max(Sl(1), Sl(2), \ldots, Sl(ps))} \]

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System Reliability Maximization for Stochastic-Flow Network Subject to Total lead-time based on Random Weighted Genetic Algorithm
Step 2. Calculate the Fitness value for each solution as follows:

Step 2.1: Generate a random number \( u_k \) in \([0,1]\) for each objective \( k \), \( k=1 \), and 2.

Step 2.2: Calculate the random weight of each objective \( k \) as \( w_k = u_k/(\sum_{i=1}^{2} u_i) \).

Step 2.3: Calculate the fitness of the solution as

\[
f(i) = w_1 \times NR_{k,1}(i) + w_2 \times NS_1(i)
\]

Step 3: Calculate the selection probability of each solution

\[
P(i) = \frac{f(i) - f_{\text{min}}}{\sum_{j=1}^{ps} (f(j) - f_{\text{min}})} \quad \text{where} \quad f_{\text{min}} = \min\{f(i), i \in \text{ps} \}.
\]

VIII. THE ENTIRE ALGORITHM BASED ON RWGA

Step 1: Set the population size (ps), the crossover rate (pc), the mutation rate (pm), and the number of generations (gn).

Step 2: Generate the initial population including successful individuals B1, B2, \ldots, Bps.

Step 3: For each individual, evaluate the network unreliability \( S_1 = 1 - R_{d,T}(B) \), and the total lead time

\[ S_2 = S_1(B). \]

Step 4: Determine the set of non-dominated solutions E and the number of non-dominated solutions NE.

Step 5: Calculate the Fitness value and the selection probability for each individual B in the current population as presented in section 5.2.

Step 6: Select parents using the selection probabilities calculated in Step 5. Apply the GA operations (described in section 5.1) to generate new populations. Apply crossover on the selected parent pairs to create N offspring. Mutate offspring with a predefined mutation rate. Copy all offspring to Pt+1 and update E if necessary.

Step 7: Randomly remove NE solutions from the new population and add the same number of solutions from E to NE.

Step 8: If the stopping condition is not satisfied, set gn=gn+1 and go to Step 5. Otherwise, return to E. After obtaining a Pareto set, change the network unreliability to be the network reliability for each Pareto solution.

IX. ILLUSTRATIVE EXAMPLES

In this section, we demonstrate the effectiveness of our proposed approach using examples taken from literature (four nodes network and six nodes network). The genetic parameters used in the proposed multi objective GA are ps = 10, Maxgen = 100, Pc = 0.95, and Pm = 0.05. The algorithm was iterated 10 times. Also, NE equals to 3, i.e. for each objective the algorithm searches the best solution and stores it as a member of E.

X. FOUR-NODES NETWORK EXAMPLE

The network has four nodes and 6 links (Figure 3), as studied by Hassan [11]. The MPs are as follows:

\( MP_1 = \{a_1, a_3\}, MP_2 = \{a_1, a_3, a_6\}, MP_3 = \{a_1, a_3, a_4, a_6\}, MP_4 = \{a_1, a_3, a_7, a_9\}, MP_5 = \{a_2, a_4, a_7\}, \text{and MP}_6 = \{a_2, a_6\}. \)

Table I lists the 8 components and associated information. Table II concisely lists the best candidate solutions for given different values for \( d, T \).

i.e. Maximum value for \( R_{d,T}(B) \) and Minimum value for \( S_1(B) \).

\[ \text{Fig 3. The four-nodes network example.} \]

<table>
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<tr>
<th>cpk</th>
<th>Capacity</th>
<th>l(cpk)</th>
</tr>
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<tbody>
<tr>
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<td>0.10</td>
</tr>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
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<td>0.10</td>
</tr>
<tr>
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</tr>
<tr>
<td>4</td>
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<td>0.90</td>
</tr>
<tr>
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<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table I. Arc capacity, probability, lead-time, and cost for the 20 available components.

<table>
<thead>
<tr>
<th>d,T</th>
<th>Rd,T(B)</th>
<th>Sl(B)</th>
<th>Assigned Components</th>
</tr>
</thead>
<tbody>
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<td>4.6</td>
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<td>9</td>
<td>8 6 7 5 2 1</td>
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<td>4.7</td>
<td>992316</td>
<td>9</td>
<td>2 6 3 5 7 8</td>
</tr>
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<td>4.8</td>
<td>994246</td>
<td>9</td>
<td>7 6 1 5 8 2</td>
</tr>
<tr>
<td>4.9</td>
<td>994358</td>
<td>11</td>
<td>1 2 7 6 3 8</td>
</tr>
</tbody>
</table>

Table II. The best candidate solutions for different values for \( d, T \).

XI. SIX-NODE NETWORK EXAMPLE

The network has six nodes and 9 links (Fig 4), as studied by Hassan, (2015). The mps are as follows: \( MP_1 = \{a_1, a_4, a_3\}, MP_2 = \{a_1, a_4, a_7, a_9\}, MP_3 = \{a_1, a_5, a_6\}, MP_4 = \{a_1, a_3, a_6, a_8\}, MP_5 = \{a_1, a_3, a_6, a_9\}, MP_6 = \{a_2, a_3, a_6\}, MP_7 = \{a_1, a_4, a_6\}, MP_8 = \{a_2, a_6, a_7, a_9\}. \)
mp₉ = {a₂, a₃, a₄, a₉}, mp₁₀ = {a₂, a₃, a₄, a₇, a₈} , and mp₁₁ = {a₂, a₃, a₅, a₈}.

Table III lists the 20 components and associated information. Table IV lists the candidate solutions found by the proposed approach for this network given d=6 and T=7. While, Table V concisely lists the best candidate solutions for different values for d, T, i.e., Maximum value for R_d,T, C(B) and minimum values for S₁(B) and C(B).

XII. DISCUSSION

Comparing the results obtained by the proposed approach to those found by the algorithm of Hassan (2015), it is observed that the values of system reliability R_d,T found by the proposed approach are better than those obtained by Hassan [11], and also the values of minimum lead-time S₁ are better than those obtained in Hassan[11]. Therefore, the proposed approach obtains better optimal solutions.

XIII. CONCLUSION

This paper investigated the component assignment component for SFN under two constraints such as system reliability and total lead-time. The purpose of this study is to maximize system reliability and minimize the total lead time. Therefore, a multi-objective components assignments problem subject to maximize system reliability and lead-time is presented and formulated as a multi-objective minimization problem. Furthermore, a multi-objective GA-based on RWGA approach is proposed to solve the presented problem. Using this proposed approach, the most optimal solution is obtained i.e., the system reliability is maximized and the total lead-time is minimized.

REFERENCES


**Table III. Arc capacity, probability, lead-time, and cost for the 20 available components**

<table>
<thead>
<tr>
<th>cpₙ</th>
<th>Capacity</th>
<th>(tcpₙ)</th>
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<td>20</td>
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**Table IV. The best optimal solutions to six nodes network.**

<table>
<thead>
<tr>
<th>d,T</th>
<th>S₀₁</th>
<th>Assigned Components</th>
<th>R_d,T</th>
<th>S₁</th>
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