

ANALYSIS ON TRANSFER FUNCTION OF SNAKE-LIKE ROBOT AND ITS STABILITY CHECKING

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Abstract - The paper discusses on the analysis of transfer function and its stability analysis on snake-like robot. Snake robots can manifold four types of motion in the surface level using different transfer function. The major assignment is to derive the transfer function of each segment in the snake-like robot body that can control the snake body to navigate with Dynamic Mechanism. The lead compensators are subject to improve the speed of response of the snake body while lag compensators optimize its errors in deviation while obstacle avoidance. This paper holds a discussion over the controller design procedure using frequency response approach and shows a comparison between the lead-lag and only lag compensators with their transient and impulse responses. Finally, the simulation results were shown to support the stability analysis of the designed control system for snake-like robot.

Keywords - Transfer Function, Snake-like Robot, Stability Analysis, Control Theory, Numerical Analysis.

I. INTRODUCTION

Snake-like robots are multi-segment machines that derive impulsion from furrow. The design of snake-like robot is organically stimulated from real snakes as snakes being able to exist in and locomote in more branches out terrains due to its style which helps to adopt diverse gaits. These characteristics of topography ability, scalability, high stability, and ground adaptability the snake-like robot can be used in numerous purposes such as pipe repairing, liberate, payload, medical purpose, space research etc. The relevance of lag-lead compensator designed using graphical domain approach is the key perception for designing the controller which consequently helps in open-handed fast response in company with small steady state error at equivalent time. Consequently, providing a high-quality control over the segmented snake robot, the stability analysis is a vital role in dynamic body. Controller design, segmentation along with wheeled approach palliates the problem that still persists in other robots referred in other like stability, obstacle avoidance in [1], interchangeability, increased fault tolerance, payload etc. A review of snake robots can be found in [2] and [3]. Most of those robots have been designed for locomotion on ground, and only a small number of functioning illustrations of swimming snake robots presently survive. The most interesting ones are the eel robot REEL II [4], the lamprey robot built at Northeastern University [5] and the spirochete-like HELIX-I [15]. A different work also carried away in [6], to fabricate an amphibious snake-like robot that can both crawl and swim for outdoor robotics tasks, taking inspiration from snakes and draw out fishes such as lampreys, and to reveal the use of central pattern generators (CPGs) as an influential method for online trajectory creation for crawling and swimming in a

real robot. An enhanced mechanical design, more powerful motors, wireless communication capabilities, onboard CPG running on a microcontroller, consequently removing the need of running the controller on an external computer. The rest of the paper is organized as follows. Section II presents the background theory and mathematical modelling of snake-like robot. Section III demonstrates the development of controller design for snake-like robot. Section IV expresses the conclusion of research works.

II. BACKGROUND THEORY AND MATHEMATICAL MODELLING OF SNAKE-LIKE ROBOT

The research of the Department of Electronic Engineering at Mandalay Technological University led to the development of snake robot, for pipe inspection purposes. The robot has the ability to pass through the pipe with rectangular cross section in order to perform pipe inspection. The next requirement is the ability of robot to transport material (e.g. cables) through the pipe. This robot should be able to pass through not only straight pipes but also curved pipes.

A. Background

Biological snakes, inchworms and caterpillars are the source of inspiration for most of the robots dealt with in this paper. Now we therefore focus more on bio-inspired imitative movements. There are several locomotion types that are the source of inspiration for Snake robots such as: Lateral undulation; Rectilinear locomotion; Sidewinding locomotion; Concertina motion and other Snake gaits. In this paper, we did build our robot based on Concertina motion. A concertina is a small accordion instrument. The name

is used in snake locomotion to indicate that the snake stretches and curves its body to move forward. The folded part is kept at a fixed position while the rest of the body is either pushed or pulled forward as shown in Fig. 1. Then, the two parts switch roles. Forward motion is obtained when the force needed to push back the fixed part of the snake body is higher than the friction forces on the moving part of the body. concertina locomotion is employed when the snake moves through narrow passages such as pipes or along branches. If the path is too narrow compared to the diameter and curving capacity of the snake, the snake is unable to locomotive [7].

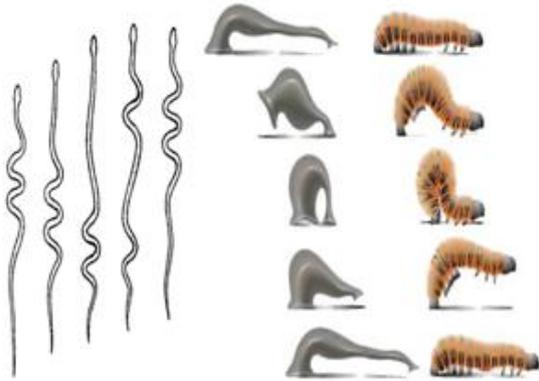


Fig.1. Concertina Motion

B. Kinematic Model

As shown in Fig. 2, let o be the origin of the absolute coordinate system, P be the point to be controlled in the head of the robot, o - xy be the absolute coordinate system. Also, let $[x_p y_p \psi_p]^T$ be the absolute coordinate of P and the orientation of the unit 1. The positions of the centre of the unit i and the joint i are defined as $[x_i y_i]^T$ and $[x_{ji} y_{ji}]^T$, respectively. Furthermore, let L_1 be the length from the front tip of each link to the centre of the unit on the link, and L_2 be the length from the centre of the rear end of the link. The joint angle ϕ_i is defined as the orientation of the unit i with respect to the unit $i-1$, and $\psi_i = \psi_p + \sum_{k=1}^{i-1} \phi_k$ ($i=1,2,3,4$) denotes the orientation of the unit i with respect to the absolute coordinate system.

Additionally, let $\dot{\theta}_i$ ($i=1, 2,3,4$) be the angular velocity of the centre of unit i . The position of the centre of the unit i is described from a geometrical relation as follows:

$$x_i = x_p + L_1 \cos \psi_p + \sum_{j=1}^{i-1} (L_2 \cos \psi_j + L_1 \cos \psi_{j+1}) \quad (1)$$

$$y_i = y_p + L_1 \sin \psi_p + \sum_{j=1}^{i-1} (L_2 \sin \psi_j + L_1 \sin \psi_{j+1}) \quad (2)$$

Where $\psi_1 = \psi_p$. Since it is assumed that the passive wheels do not slip sideways, we need to take into account the velocity constraint condition.

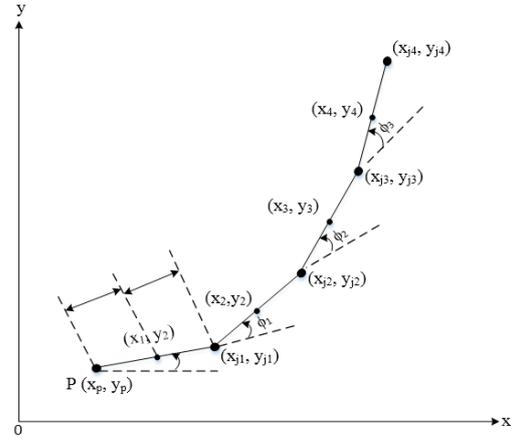


Fig.2. Definition of Coordinate Variables

The detailed derivation for Equation (1) and (2) could be evaluated as follows:

$$x_i = x_p + L_1 \cos \psi_1 + L_2 \cos \psi_1 + L_1 \cos \psi_2 + L_2 \cos \psi_2 +$$

$$L_1 \cos \psi_3 + L_2 \cos \psi_3 + L_1 \cos \psi_4$$

$$y_i = y_p + L_1 \sin \psi_1 + L_2 \sin \psi_1 + L_1 \sin \psi_2 + L_2 \sin \psi_2 +$$

$$L_1 \sin \psi_3 + L_2 \sin \psi_3 + L_1 \sin \psi_4$$

$$\dot{x}_i \cos(\alpha_i + \psi_i) + \dot{y}_i \sin(\alpha_i + \psi_i) + r \dot{\theta}_i \sin \alpha_i = 0$$

$$A \dot{\xi} = B u$$

$$\xi = [x_p y_p \psi_p \phi_1 \phi_2 \phi_3]^T$$

$$u = [\dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 \dot{\theta}_4 \dot{\phi}_1 \dot{\phi}_2 \dot{\phi}_3]^T$$

$$A_{11} := \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}, \quad A_{12} := \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & a_{32} & 0 \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

$$B_1 := -r \text{diag}(\sin \alpha_1, \sin \alpha_1, \sin \alpha_1, \sin \alpha_1)$$

$$a_{11} := \cos(\alpha_1 + \psi_1)$$

$$a_{12} := \sin(\alpha_1 + \psi_1)$$

$$a_{13} := L_1 \sin \alpha_1$$

$$a_{23} := L \sin(\alpha_2 + \phi_1) + L_1 \sin \alpha_2$$

$$a_{24} := L_1 \sin \alpha_2$$

$$a_{33} := L \sin(\alpha_3 + \phi_1 + \phi_2) + a_{34}$$

$$a_{34} := L \sin(\alpha_3 + \phi_2) + L_1 \sin \alpha_3$$

$$a_{35} := L_1 \sin \alpha_3$$

$$a_{43} := L \sin(\alpha_4 + \phi_1 + \phi_2 + \phi_3) + a_{44}$$

$$a_{44} := L \sin(\alpha_4 + \phi_2 + \phi_3) + a_{45}$$

$$a_{45} := L \sin(\alpha_4 + \phi_3) + L_1 \sin \alpha_4$$

$$a_{46} := L_1 \sin \alpha_4$$

$$L := L_1 + L_2$$

$$L_1 = 0.103[\text{m}], L_2 = 0.123[\text{m}], r = 0.075[\text{m}],$$

$$\alpha_i = -\frac{\pi}{4} [\text{rad}] (i=1,3),$$

$$\alpha_i = \frac{\pi}{4} [\text{rad}] (i=2,4)$$

$$\dot{x}_i \cos(-\alpha_i + \psi_i) + \dot{y}_i \sin(-\alpha_i + \psi_i) - r \dot{\theta}_i \sin(-\alpha_i) = 0$$

$$\dot{x}_i \cos \psi_i + \dot{y}_i \sin \psi_i + r \dot{\theta}_i \tan \alpha_i = 0$$

$$\dot{x}_i \sin \psi_i - \dot{y}_i \cos \psi_i = 0$$

III. IMPLEMENTATION OF CONTROLLER DESIGN

Two different methods for controller design are discussed in [8]. Design of only lead compensator may satisfy the speed of response of the snake body which may affect the wireless control of the snake robot described in [9] and [10]. An optimized control of snake robot needs a lag-lead compensator to be designed for the following specification.

- The controlled output of the system (motor) with the given specification of maximum peak of 25%
- a rise time of 0.3 seconds
- No.of segments to be present in the multi-segment robot is 4.

In this work 1.2-watt DC motor has been used which is smaller in size and suitable for the desired snake-like robot. Table I shows detailed specification of this motor.

Table I Technical Specification of DC Motor 1.2 watt

Parameter	Value
Armature Inductance (L_a)	0.223 mH
Armature Resistance (R_a)	11 Ohm
Rotor Inertia (J_m)	0.306 gcm ²
Torque Constant (k_t)	5.08 mNm/A
No Load Speed	11100 rpm
No Load Current	10.4 mA
Speed/Torque Gradient	4050 rpm/mNm

Applying a constant stator current and assuming magnetic linearity, the basic motor equations are

$$T_m = K I \quad (2)$$

$$e_a = K_m \omega \quad (3)$$

Let the switch SW be closed at $t = 0$. After the switch is closed,

$$V_t = e_a + I_a R_a + L_{aq} \frac{di_a}{dt} \quad (4)$$

From Equation (3) and (4)

$$V_t = K_m \omega + I_a R_a + L_{aq} \frac{di_a}{dt} \quad (5)$$

$$T_m = K_m I_a = J \frac{d\omega}{dt} + B\omega \quad (6)$$

The term $B\omega$ represents the rotational loss torque of the system. The state-space representation is given by the equations:

$$\dot{x} = Ax + Bu \quad (7)$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} \omega \\ i_a \end{bmatrix}, A = \begin{bmatrix} -B/J & K_m/J \\ -R_a/L_{aq} & -K_m/L_{aq} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1/J \\ 1/L_{aq} & 0 \end{bmatrix}, u = \begin{bmatrix} V_t \\ T_L \end{bmatrix}$$

$$C = [1 \ 0], D = [0]$$

The transfer function between angular velocity and voltage is

$$\frac{\omega}{V_t} = \frac{A_v}{s^2 + B_1 s + B_0} \quad (8)$$

The transfer function between angular velocity and Load Torque is

$$\frac{\omega}{T_L} = \frac{A_L}{s^2 + B_1 s + B_0} \quad (9)$$

where

$$A_v = \frac{K_m}{J L_{aq}}$$

$$A_L = -\frac{K_m}{J^2}$$

$$B_1 = \frac{B}{J} + \frac{K_m}{L_{aq}}$$

$$B_0 = \frac{(B + R_a) K_m}{J L_{aq}}$$

In most DC motors, the rotor inductance and the value of B are small that can be neglected to lead to reduced order. If L_{aq} is neglected then Equation (9) becomes

$$V_t = K_m \omega + I_a R_a \quad (10)$$

If B is neglected then Equation (10) becomes

$$T_m = K_m I_a = J \frac{d\omega}{dt} + T_L \quad (11)$$

The current-voltage relationship for the left hand side of the equation can be written and manipulated to relate between voltage and angular velocity.

$$I_a = \frac{V_t - e_a}{R_a} \quad (12)$$

$$\frac{T_m}{K_m} = \frac{V_t - K_m \omega}{R_a} \quad (13)$$

$$\frac{J \frac{d\omega}{dt} + T_L}{K_m} = \frac{V_t - K_m \omega}{R_a} \quad (14)$$

$$\frac{d\omega}{dt} + \left(\frac{K_m^2}{J R_a} \right) \omega = \left(\frac{K_m}{J R_a} \right) V_t - \frac{T_L}{J} \quad (15)$$

The detailed derivation was shown in above. The final transfer function is as follows. The motor transfer function obtained for the uncompensated system is:

$$TF = \frac{k_t}{J L s^2 + (J R + B L) s + (B R + k_e k_t)}$$

$$TF = \frac{5.08}{(0.06802) s^3 + (0.06802) s^2 + (7.338) s}$$

This work bears the importance of designing and distinguishing lag and lead-lag compensators designed for each segment of a 3-segmented snake robot.

The lag compensators for three-segments could be derived from the DC motor's mathematical model and the following are accepted control system transfer function with lag compensators.

For lag compensator (1),

$$T(s) = \frac{12.36s + 1.743}{0.06802s^4 + 3.454s^3 + 7.396s^2 + 12.49s + 1.743}$$

For lag compensator (2),

$$T(s) = e^{-0.333s} \frac{12.36s + 1.743}{0.06802s^4 + 3.454s^3 + 7.396s^2 + 12.49s + 1.743}$$

For lag compensator (3),

$$T(s) = e^{-0.667s} \frac{12.36s + 1.743}{0.06802s^4 + 3.454s^3 + 7.396s^2 + 12.49s + 1.743}$$

The lag-lead compensators for three-segments could be derived from the DC motor's mathematical model and the following are accepted control system transfer function with lag-lead compensators.

For lag-lead compensator (1),

$$T(s) = \frac{104s^2 + 208.7s + 104.5}{0.06802s^5 + 3.818s^4 + 25.94s^3 + 146.9s^2 + 216.1s + 104.5}$$

For lag-lead compensator (2),

$$T(s) = e^{-0.333s} \frac{104s^2 + 208.7s + 104.5}{0.06802s^5 + 3.818s^4 + 25.94s^3 + 146.9s^2 + 216.1s + 104.5}$$

For lag-lead compensator (3),

$$T(s) = e^{-0.667s} \frac{104s^2 + 208.7s + 104.5}{0.06802s^5 + 3.818s^4 + 25.94s^3 + 146.9s^2 + 216.1s + 104.5}$$

IV. SIMULATION RESULTS

The simulation results for stability analysis on snake-like robot with two types of compensators like lag compensators and lag-lead compensators have been evaluated with the help of MATLAB. At first, the uncompensated snake-like robot has to be developed. And then, the control systems with lag compensator have to be evaluated and the stability analyses of control systems with lag-lead compensators have to be developed in this section.

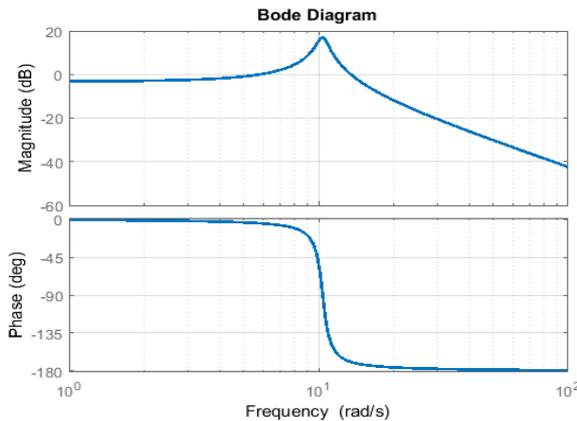


Fig.3. Simulation Results of Uncompensated Control System for Snake-like Robot

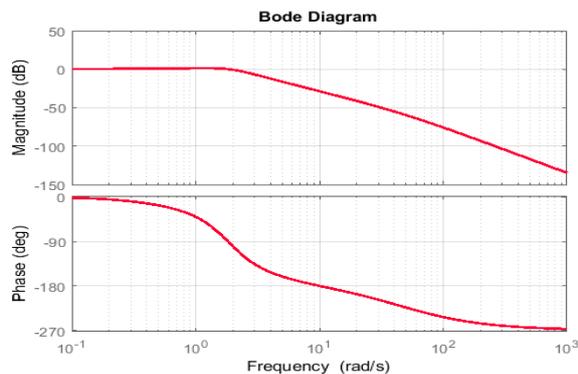


Fig.4. Bode Result for Control System with Lag Compensator 1 Condition

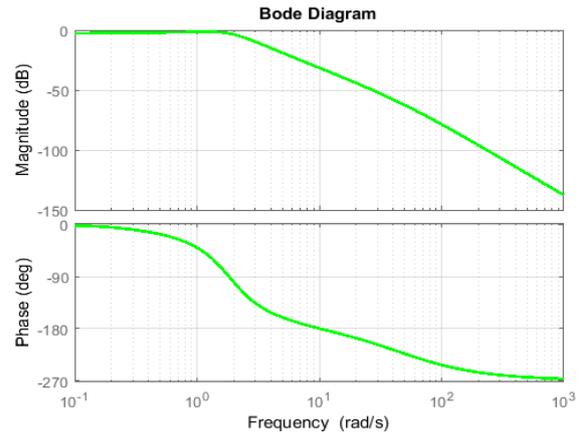


Fig.5. Bode Result for Control System with Lag Compensator 2 Condition

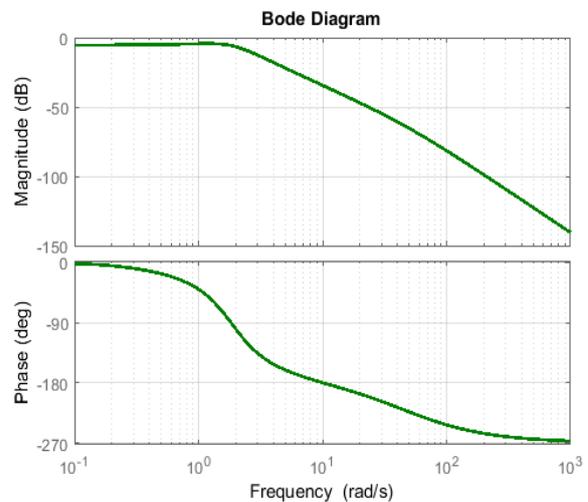


Fig.6. Bode Result for Control System with Lag Compensator 3 Condition

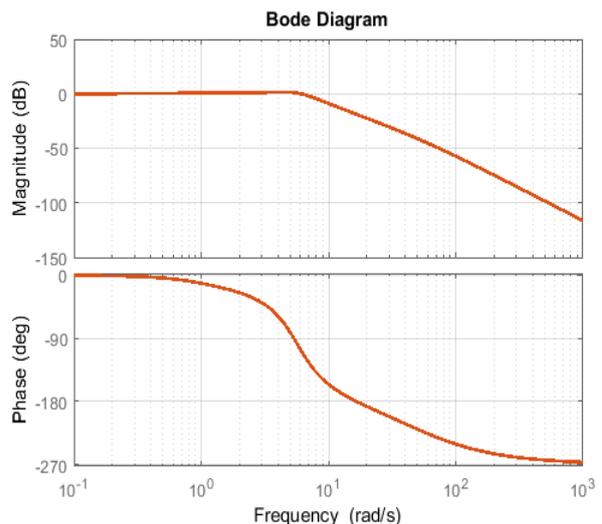


Fig.7. Bode Result for Control System with Lag-Lead Compensator 1 Condition

Fig.3 shows the uncompensated control system stability checks for snake-like robot system. Fig.4 to 6 shows the Bode Result for Control System with Lag Compensator 1,2 and 3 Conditions. Fig. 7 to 9 shows the Bode Result for Control System with Lag-Lead

Compensator 1, 2 and 3 Conditions. Based on the simulation results on Bode analysis for snake-like robot, the lag compensator could not have affected on the uncompensated snake-like robot system but the lag-lead compensator confirmed that the stability of snake-like robot system.

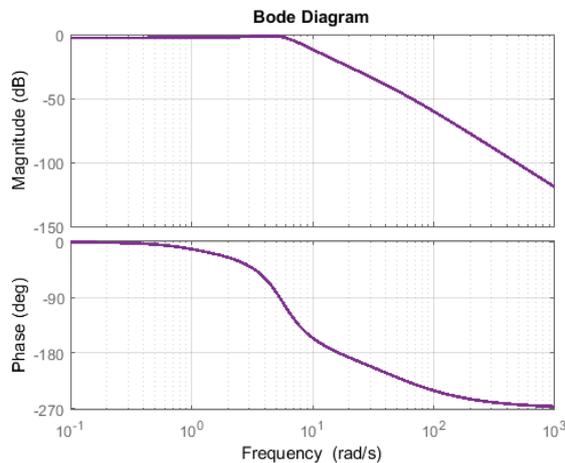


Fig.8. Bode Result for Control System with Lag-Lead Compensator 2 Condition

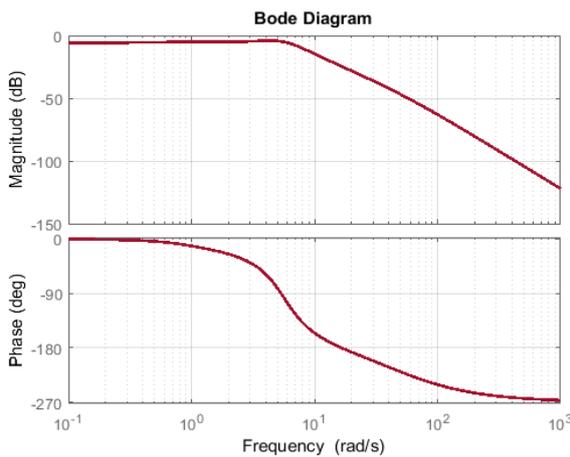


Fig.9. Bode Result for Control System with Lag-Lead Compensator 3 Condition

V. CONCLUSIONS

Accordingly, from the simulation results, the lag-lead compensator designed using frequency domain approach compensates the system to a quite good extent by minimizing the steady state error and by giving a swift response relative to the uncompensated as well as lag compensated system. Particularly the response time is less in comparison to the lag compensated system designed earlier. As well as the bode responses for phase shifted linear simulation

results show each joints to be at phase difference to each other resulting in serpentine motion curve. The obstacles are easily overcome by increasing the initial phase of the input signal. The modular along with the wheeled approach helps to follow the motion curve using differential friction with reduced side slipping and incase of any malfunctioning it can easily be interchanged.

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