BER PERFORMANCE ANALYSIS OF AN ADAPTIVE RECEIVER WITH IMPERFECT CHANNEL ESTIMATION OVER HOYT FADING CHANNELS

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Abstract - Analysis of Bit Error Rate (BER) has been carried out for Adaptive Continuous Rate Multilevel-Quadrature Amplitude Modulation (ACR M-QAM) over Hoyt flat fading channels considering imperfect channel estimation at the receiver side with adaptive transmission techniques. In this analysis the error due to the time delay in the feedback path has been considered. However, the same analysis is valid for any other type of estimation error which may occur in the receiver and degrades the performance of the wireless receiver. In this analysis it has been noticed that the BER is directly related to the amount of correlation between the actual channel gain and the estimated gain. It has been observed from the analysis that the performance of the receiver starts degrading if the correlation between the actual and the estimated channel gain is less than 0.99 and it almost unable to detect if correlation decreases to 0.81.

Index Terms - Bit Error Rate, MQAM, Time Delay, Imperfect Channel Estimation, Hoyt Fading Channel.

I. INTRODUCTION

Channel estimation at the receiver side is important to enhance the performance of communication systems particularly in wireless communication systems. In a wireless system, a channel estimator is applied at the receiver to estimate the channel condition and the channel information is feedback to the transmitter through a lossless path.

However, the channel estimation in a real time is challenging and there is a possibility of an imperfect estimation. Researchers have studied this topic in various communication models for more than a decade and presented in the literature [1]-[15]. In [3], adaptive modulation and its impact, over a Nakagami-m fading channel has been studied considering ACRand ADR (Adaptive Discrete Rate) MQAM. The impact of time delay for an adaptive receiver over the same fading channel also has been studied here.

A capacity analysis has been borne out in [4] for Optimal and Sub Optimal Power and Rate Adaptation Techniques considering an Imperfect Channel Estimation for Multilevel-Quadrature Amplitude Modulation (M-QAM) over a Rayleigh fading channel. In [5] the issues of ISI, time-varying, and multiple accesses in the context of an error about the channel measurement available at the receiver have been considered. In the same work it has also been shown that the time variation of the channel and the error on the estimate of the channel are tightly linked. In [6] the impact of imperfect channel estimation on the Variable-Rate Variable-Power QAM (Quadrature Amplitude Modulation) performance is contemplated for a flat fading environment. Here, a set of new analytical expression is derived that shows the high sensitivity of the BER (Bit Error Rate) to both the estimated MSE (Mean Square Error) and the system adaptation delay.

Similarly, a general approach to calculate the exact BER of M-QAM with the PSAM (Pilot-Symbol Assisted Modulation) in flat Rayleigh fading channels is given in [7] when there are some channel estimation errors. In [8] the authors propose an adaptive multi-mode transmission strategy to improve the spectral efficiency achieved in the multiple-input multiple-output (MIMO) broadcast channel with imperfect channel state information. The adaptive strategy adjusts the number of active users, denoted as the transmission mode, to balance, the transmit array gain, the spatial division multiplexing gain, and the residual inter-user interference.

The optimum power profile and the ergodic capacity have been derived for Rayleigh fading channels with respect to an average or a peak transmit power, along with more realistic interference outage constraints in [9]. Also, the impact of channel estimation quality on the ergodic capacity has been highlighted. The performance analysis of a space-time coded MIMO system with the Variable-Rate-Adaptive Modulation over flat Rayleigh fading channels for both perfect and imperfect channel state information (CSI) has been presented in [10]. In [11] the ergodic capacity of bidirectional amplify-and-forward relay selection network has been analyzed. Also, the imperfect CSI takes into account, which includes outdated CSI and channel estimation error, caused by the time-variation of the channel and the imperfect channel estimation. A system where the receiver should harvest energy from the transmitter by wireless energy transfer to bear out its wireless information transmission has been studied in [12].
In [13] an optimal precoding method for a multiple antenna relay node is investigated in order to maximize
the achievable rate of the cooperative communication system. It is assumed that only the channel covariance
matrices of the relays receive and transmit channels are available for the relay and that the antennas of the relay
are correlated. The optimization of an amplify and forward (AF) relay network with time delay and
estimation error in the channel state information (CSI) has been modeled by the channel time variation and the
stochastic error, respectively in [14]. The MIMO channel capacity for dual and asymptotic case over
hoyt fading channel is studied in [15]. In the literature study we found that the BER analysis with estimation
error has mainly being analysed only for the homogeneous fading channels only. This generates a
motive to carry out the analysis for non homogeneous fading channels like Hoyt. The Hoyt fading channels
[2] receive attention of researchers due to its flexibility, simple analytical form and also because it fits best into
the practically obtained data. More over, the ACR-MQAM technique is also has been considered
here, where the constellation size is adapted with the change of channel and the receive signal SNR . In this
paper, Bit Error Rate analysis of a single antenna system is done, for an imperfect channel knowledge
considering ACR-MQAM over the Hoyt fading channels. The paper is organized as follows: In section
II the channel and system model is presented. In section III the BER over imperfect Hoyt Fading is
derived. In section IV the numerical results and analysis is done. Conclusions are given in section V.

II. CHANNEL AND SYSTEM MODEL

![Fig 1: Adaptive System Model](image)

An adaptive transmission scheme has been considered for analysis and is shown in Fig.1. The transmitter
shown in the system model is used to send an input message ‘w’. The encoded codeword for ‘w’ is ‘x’ and
it is transmitted over the time-varying Hoyt channel as x[i] at time i. Since the channel is changing with the
time, then the channel gain h[i] changes over the transmission of the code as well.

The two sided power spectral density of the additive noise n(i) has been assumed as 2N0. Designing a
perfect channel estimator is always challenging task, which may lead to introduce of small amount of error
in the estimation. Also, due to the delay in the feeding

path ( τfb ) may introduce error in the channel
information provided to the transmitter by a channel
estimation. Hence, we consider h(i) is the estimated
and delayed version of the channel gain h (i) and
there may present little difference between this two
parameter due to reason mention mentioned above.
The average power to be transmitted is denoted by S .

For a constant transmitted power S the instantaneous
SNR at the receiver will be

\[\gamma(i) = \frac{|h(i)|^2}{N_0 B} \]

where B is the received signal bandwidth. In the considered
model, the constellation size selector will adaptively
change its constellation size depends on the receiver
signal SNR so that the modulator and demodulator
must be configured at any instant with same
collection size.

Again, for a system design the constellation size is
based on channel estimation at time i, but actually the
data are sent over the time i+τ, where τ is the delay in
communication. For a large τfb, this considered
model will not work properly. So, here it is assumed
that τfb ≤ τ with a minimum rate of system
constellation reconfiguration. On the other hand,
transmitter adaptively adjust its power based on
feedback hence, instantaneous transmit power at time i
is a function of h(i) (or h(i)). In the considered
model the channel has been assumed to be slow,
frequency nonselective, with Hoyt fading statistics.

Over one bit duration Ti , the complex low pass
equivalent of the signal received by the antenna can be
expressed as,

\[r(t) = \alpha e^{j\varphi} s(t) + n(t) \]  

where, s(t) is the transmitted bit with energy En .

Random variable (RV) ‘ \varphi ’ represents the phase and
\alpha is the Hoyt distributed fading amplitude having
PDF given by [2],

\[p(\alpha) = \frac{(1 + q^2)\alpha}{q\Omega} e^{-\frac{(1+q^2)\alpha^2}{4\Omega^2}} I_0 \left( \frac{(1-q^2)\alpha^2}{4q^2\Omega^2} \right) \]  

where, \(\Omega = E(\alpha^2)\cdot q \in [0,1]\) is the Hoyt fading
parameters and \(I_0 \) is the modified Bessel function of
the first kind and zeroth order. The SNR pdf of Hoyt
fading channel is shown in [2,11][2].

III. BER OVER IMPERFECT HOYT FADEING CHANNEL

To derive the BER performance of the system
described in the previous section with imperfect
channel estimations over the Hoyt fading channels
requires the knowledge of correlated joint pdf of the
PDF given by [2],

\[p(\alpha) = \frac{(1 + q^2)\alpha}{q\Omega} e^{-\frac{(1+q^2)\alpha^2}{4\Omega^2}} I_0 \left( \frac{(1-q^2)\alpha^2}{4q^2\Omega^2} \right) \]  

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the first kind and zeroth order. The SNR pdf of Hoyt
fading channel is shown in [2,11][2].
Hoyt distribution. Hence, the analysis has been carried out in two different sections. In the first section the derivation of correlated joint pdf is given and in the second section we present the bit error rate analysis.

A. Derivation of Joint PDF of Hoyt with Correlation

Using the Hoyt fading model in [15] the square of the Hoyt distribution can be written in terms of Gaussian distribution as,

$$\alpha_i^2 = X_i^2 + Y_i^2, (i=1,2) \quad (3)$$

where ‘$X_i$’ and ‘$Y_i$’ are independent zero mean Gaussian RVs with variances $\sigma_x^2$ and $\sigma_y^2$, respectively. In this representation the Hoyt RV $\alpha_i$ has the pdf given in (2) where the fading parameter $q = \frac{\sigma_x}{\sigma_y}$. Assuming $X_1(Y_1)$ and $X_2(Y_2)$ are correlated with correlation coefficient ‘$\rho$’, it can be shown that RVs $\alpha_1$ and $\alpha_2$ are correlated with correlation coefficient $\rho$. From (3) the joint CF of bivariate RVs $\alpha_1^2$ and $\alpha_2^2$ can be expressed as,

$$\Phi_{\alpha_1^2, \alpha_2^2}(j \omega_1, j \omega_2) = \Phi_{X_1^2, X_2^2}(j \omega_1, j \omega_2) \Phi_{Y_1^2, Y_2^2}(j \omega_1, j \omega_2) \quad (4)$$

where $\Phi_{h_1,h_2}(h_1, h_2)$ is the joint CF of $h_1$ and $h_2$. An expression for the CF of $\Phi_{X_1^2, X_2^2}(j \omega_1, j \omega_2) \Phi_{Y_1^2, Y_2^2}(j \omega_1, j \omega_2)$ of bivariate Gaussian RVs $X_1(Y_1)$ and $X_2(Y_2)$ can be obtained from the joint density function in [16]. Thus, the joint CF in can be expressed as,

$$\Phi_{\alpha_1^2, \alpha_2^2}(j \omega_1, j \omega_2) = \frac{1}{4 \sigma_x \sigma_y \sqrt{1 - \rho^2}} \sum_{n=0}^{\infty} \frac{\rho^n}{(2k-1)!!(2l-1)!!} \left( \frac{1}{2(1-\rho^2)\sigma_x^2} + j \omega_1 \right)^k \left( \frac{1}{2(1-\rho^2)\sigma_y^2} + j \omega_2 \right)^l \quad (5)$$

(5) In this paper, for the convenience of presentation but without loss of generality, we assume $\sigma_x^2 = 1$ resulting in $\sigma_y^2 = q^2$. From (5), taking its inverse Fourier transform followed by a RV transformation corresponding to multiplying by a factor $E_b / N_0$ the joint PDF of input SNR $\gamma_1$ and $\gamma_2$ can be obtained as,

$$f_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) = \frac{1}{4 \sigma_x \sigma_y \sqrt{1 - \rho^2}} E_b \sum_{n=0}^{\infty} \frac{\rho^n}{(2k-1)!!(2l-1)!!} \frac{1}{k! \rho^{k+l} \Gamma(k+l+1)} \left( \frac{1}{2(1-\rho^2)\sigma_x^2} + j \omega_1 \right)^k \left( \frac{1}{2(1-\rho^2)\sigma_y^2} + j \omega_2 \right)^l \quad \text{for } \gamma_1 \geq 0, \gamma_2 \geq 0 \quad (6)$$

where, $\frac{N_0}{E_b} = 1 + \frac{q^2}{\sigma_x^2}$

Simplifying (6), the joint pdf can be expressed as,

$$f_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) = \frac{1}{4 \sigma_x \sigma_y \sqrt{1 - \rho^2}} E_b \sum_{n=0}^{\infty} \frac{\rho^n}{(2k-1)!!(2l-1)!!} \frac{1}{k! \rho^{k+l} \Gamma(k+l+1)} \left( \frac{1}{2(1-\rho^2)\sigma_x^2} + j \omega_1 \right)^k \left( \frac{1}{2(1-\rho^2)\sigma_y^2} + j \omega_2 \right)^l \quad \text{for } \gamma_1 \geq 0, \gamma_2 \geq 0 \quad (7)$$

B. Bit Error Rate Analysis:

To find the BER over an imperfect Hoyt fading channel we have to know the corresponding conditional probability. The relation between joint and conditional probability is given as,

$$f_{r_1 | r_2}(r_2, r_1) = \frac{f_{r_1, r_2}(r_2, r_1)}{f_{r_2}(r_2)} \quad (8)$$

Now putting (7) and the value of $f_{r_1}(r_1)$ as given in ([2.11],2) in (8), the expression for conditional pdf can be obtained as,

$$f_{r_1 | r_2}(r_2, r_1) = \frac{1}{4 \sigma_x \sigma_y \sqrt{1 - \rho^2}} E_b \sum_{n=0}^{\infty} \frac{\rho^n}{(2k-1)!!(2l-1)!!} \frac{1}{k! \rho^{k+l} \Gamma(k+l+1)} \left( \frac{1}{2(1-\rho^2)\sigma_x^2} + j \omega_1 \right)^k \left( \frac{1}{2(1-\rho^2)\sigma_y^2} + j \omega_2 \right)^l \quad \text{for } r_1 \geq 0, r_2 \geq 0 \quad (9)$$

For MQAM the BER($\gamma_1$) can be calculated as [3],

$$\text{BER}(\gamma_1) = \int_0^\infty \text{BER}(r_2 | \gamma_1) \rho_{\gamma_2} \frac{1}{\gamma_2} d\gamma_2$$

(10)
where, $BER(\gamma_i/\gamma_1) = 0.2(5BER_0)^{\gamma_i/\gamma_1}$ and the constant $k_0 = -\ln(5BER_0)$. Which imply $5BER_0 = e^{-k_0}$. So, $(10)$ can be expressed as,

$$BER(\gamma_i) = \frac{2}{0.2} \sum_{0}^{\gamma_i} (1-q)^{\gamma_i}f_{\gamma_i}(\gamma_i) d\gamma_i$$

(11)

Putting $(9)$ and the value of $BER(\gamma_i/\gamma_1)$ in $(11)$ and simplifying the equation, the equation can be written as,

$$BER(\gamma_i) = \frac{2}{0.2} \sum_{0}^{\gamma_i} (1-q)^{\gamma_i}f_{\gamma_i}(\gamma_i) d\gamma_i$$

(12)

Now the corresponding BER for an imperfect Hoyt Fading channel can be calculated using the formula given in [3] as,

$$BER = \int_{0}^{\gamma_i} BER(\gamma_i)f(\gamma_i) d\gamma_i$$

(13)

Now putting $(12)$ and $f_{\gamma_i}(\gamma_i)$ in $(13)$, and solving the equation the BER for an imperfect Hoyt channel estimation over an adaptive receiver can be expressed in the close form of equation as,

$$BER = \frac{2(1-q)^{\gamma_i} \sum_{0}^{\gamma_i} \sum_{0}^{\gamma_i} \sum_{0}^{\gamma_i} \sum_{0}^{\gamma_i} (k+1+\gamma_i)^{\gamma_i} \gamma_i^2}{2(1-q)^{\gamma_i} \sum_{0}^{\gamma_i} \sum_{0}^{\gamma_i} \sum_{0}^{\gamma_i} \sum_{0}^{\gamma_i} (k+1+\gamma_i)^{\gamma_i} \gamma_i^2}$$

(14)

In this analysis the ACR-MQAM (Adaptive Continuous Rate-MQAM) has been considered for all SNR as given in [30], [42] which indicate the upper bound of average BER degradation.

**IV. RESULTS AND ANALYSIS**

Fig 2 and 3 shows the BER for imperfect channel estimation as a function of normalized time delay for different $q$ values of the Hoyt fading parameter. Here we consider two different case for targeted BER ($BER_0$) $10^{-2}$ and $10^{-6}$. From the fig:2 and 3 it can be observed that for a imperfect channel, normalized time delay up to $10^{-2}$ can be tolerated without any noticeable degradation of average BER. But in both the cases for decrease of $q$ which indicate more faded condition a notable increase of BER can be seen. It has been observed from the analysis that the performance of the receiver starts degrading if the correlation between the actual and the estimated channel gain is less than 0.99 and it almost unable to detect if correlation decreases to 0.81.

**REFERENCES**

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