THE EFFECTS OF OFDM SYNCHRONIZATION IMPERFECTIONS AND CALCULATION OF UNCODED BERS FOR THE IDEALIZED OFDM SYSTEM MODEL

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Abstract - The aim of this paper is to provide some theoretical background on the OFDM transmission technique. We review the block diagram of a “classic” OFDM system, which employs a GI to mitigate the impairments of the multipath radio channel. We also discuss several design considerations related to hardware properties and derive the mathematical model for an idealized system, leading to the conclusion that data symbols can be transmitted independently of each other [i.e., without ISI and intercarrier interference (ICI)]. Moreover, the effects of synchronization imperfections are analyzed, like carrier frequency and phase offsets and timing errors. Differential and coherent detection schemes can be evaluated for Rayleigh and Rician fading channels. We also show, that for the system proposal under investigation, differential detection in frequency direction is much preferable to differential detection in frequency direction. Imperfect synchronization and channel estimation may be assessed by extending the system model used and by incorporating the SNR degradations due to ICI and ISI. Basic aspects are discussed in this paper. Issues for further refinement of the methods are addressed.

Keywords - frequency-division multiplexing (FDM), Signal carrier (SC), bit error rate (BER), Signal-to-noise ratio (SNR), Inter-symbol interference (ISI), Binary phase-shift keying (BPSK), Quadrature Phase Shift Keying (QPSK)

I. INTRODUCTION

OFDM is a parallel transmission scheme, where a high-rate serial data stream is split up into a set of low-rate substreams, each of which is modulated on a separate SC (FDM). Thereby, the bandwidth of the SCs becomes small compared with the coherence bandwidth of the channel; that is, the individual SCs experience flat fading, which allows for simple equalization. This implies that the symbol period of the substreams is made long compared to the delay spread of the time-dispersive radio channel.

Calculation of uncoded BERs for the idealized OFDM system model is done. This method is largely based on work presented in [1]. Differential and coherent detection schemes can be evaluated for Rayleigh and Rician fading channels. We also show, that for the system proposal under investigation, differential detection in time direction is much preferable to differential detection in frequency direction.

II. OFDM INTRODUCTION AND SYSTEM MODEL

By selecting a special set of (orthogonal) carrier frequencies, high spectral efficiency is obtained because the spectra of the SCs overlap, while mutual influence among the SCs can be avoided. The derivation of the system model shows that by introducing a cyclic prefix (the GI), the orthogonality can be maintained over a dispersive channel. Figure 1 shows the block diagram of a simplex point-to-point transmission system using OFDM and FEC coding. The three main principles incorporated are as follows: The IDFT and the DFT are used for, respectively, modulating and demodulating the data constellations on the orthogonal SCs [2]. These signal processing algorithms replace the banks of I/Q-modulators and demodulators that would otherwise be required.

Figure 1 Simplex point-to-point transmission using OFDM.

The second key principle is the introduction of a cyclic prefix as a GI, whose length should exceed the maximum excess delay of the multipath propagation channel [3]. Due to the cyclic prefix, the transmitted signal becomes periodic, and the effect of the time-dispersive multipath channel becomes equivalent to a cyclic convolution, discarding the GI at the receiver. Due to the properties of the cyclic convolution, the effect of the multipath channel is limited to a pointwise multiplication of the transmitted data constellations by the channel TF, or the FT of the channel IR; that is, the SCs remain orthogonal.

III. OFDM SYSTEM MODEL

Mathematically, the OFDM signal is expressed as a sum of the prototype pulses shifted in the time and...
frequency directions and multiplied by the data symbols. In continuous-time notation, the kth OFDM symbol is written
\[ s_{RF,k}(t - kT) = \frac{1}{T_{FFT}} \sum_{l=-\infty}^{\infty} x_{ik} e^{(2\pi f_c + \frac{j2\pi}{T_{FFT}})(t-kT)} \]

The impact of an FFT timing offset at the receiver can be analyzed mathematically by shifting the integration interval of the matched filter bank. For a timing error of \( \delta t \), the ideal interval \( t \in [kT, kT + T_{FFT}] \) becomes \( t \in [kT + \delta t, kT + T_{FFT} + \delta t] \) and is written
\[ y_{l,k} = \frac{1}{T_{FFT}} \int_{kT}^{kT+T_{FFT}} r(t) e^{j2\pi f_c (t-kT)} e^{-j2\pi \delta t/T_{FFT}} dt = e^{j2\pi \delta t/T_{FFT}} \int_{kT}^{kT+T_{FFT}} h(t) s(t-t) dt + n(t) \]

where \( \delta t \) is assumed to be sufficiently small (typically \( \delta t < T_{guard} \)) that no ISI arises due to the timing error. In other words, the error is small enough for the channel IR to remain within the GI. Therefore, the receiver window still does not overlap with the preceding or consecutive OFDM symbol; that is, no energy is collected from these adjacent OFDM symbols, and the demodulated signal can be expressed from the transmitted symbol \( s_k(t) \) again. Following the same steps we obtain for the second part of with \( u = T_{FFT} + \delta t \).

**IV. SYNCHRONIZATION ERRORS**

As an introduction to the work on synchronization algorithms, this topic reviews the relevant effects of synchronization errors. Original work on this topic is found in numerous publications. A comprehensive overview is given in [4].

**A. FFT Time Synchronization Error**

The impact of an FFT timing offset at the receiver can be analyzed mathematically by shifting the integration interval of the matched filter bank. For a timing error of \( \delta t \), the ideal interval \( t \in [kT, kT + T_{FFT}] \) becomes \( t \in [kT + \delta t, kT + T_{FFT} + \delta t] \) and is written
\[ y_{l,k} = \frac{1}{T_{FFT}} \int_{kT}^{kT+T_{FFT}} r(t) e^{j2\pi f_c (t-kT)} e^{-j2\pi \delta t/T_{FFT}} dt = e^{j2\pi \delta t/T_{FFT}} \int_{kT}^{kT+T_{FFT}} h(t) s(t-t) dt + n(t) \]

Moving the term \( e^{-j2\pi \delta t/T_{FFT}} \) out of the integral yields the expression for the demodulated signal constellations in case of a timing error,
\[ y_{l,k} = x_{ik} h_{ik} e^{-j2\pi \delta t/T_{FFT}} + n_{ik} + n_{ik} \]

where \( \delta t' \) is the timing offset in samples. It is evident that a timing offset gives rise to a progressive phase rotation of the signal constellations. The phase rotation is zero at the center frequency, and it linearly increases toward the edges of the frequency band. It is easily verified from that a timing offset in one sample introduces a phase shift of \( \pm \pi \) to the outermost SCs (having \( i \equiv \pm N/2 \)), regardless of the FFT length. In Figure 3, this effect is visualized for a 64-carrier OFDM system with zero carriers at fc and at the edges of the frequency band.
B. Carrier Synchronization Error

Frequency offsets are typically introduced by a (small) frequency mismatch in the local oscillators of the transmitter and the receiver. Doppler shifts can be neglected in indoor environments.

The impact of a frequency error can be seen as an error in the frequency instants, where the received signal is sampled during demodulation by the FFT. Figure 4 depicts this twofold effect. The amplitude of the desired SC is reduced (“+”), and ICI arises from the adjacent SCs (“□”).

\[ r'(t) = r(t)e^{j2\pi ft + \theta} \]  
Eq 7

Mathematically, a carrier offset can be accounted for by a frequency shift \( \delta f \) and a phase offset \( \theta \) in the lowpass equivalent received signal

With (5.9) we obtain

\[
y_{ik} = \sum_{n=-\infty}^{\infty} x_{kn} \frac{1}{\tau(f)} \int_{t-i\delta t}^{t+i\delta t} h(t) e^{j2\pi nF_{FFT}f} dt + n_{ik}
\]  
Eq 8

Figure 4 Phase rotation due to a carrier offset of 1/16 of the SC spacing. The received signal constellations distorted by ICI are shown.

C. Common Carrier and Timing Offset

Evaluating the above expressions for simultaneous timing (\( \delta t \)), frequency [\( \delta f \), \( \delta f_i = \text{round}(\delta f / F) \)] and phase (\( \theta \)) offsets, the system model for the generalized case is obtained. It is written as

\[
y_{i+\delta t,ik} = x_{ik} h_{ik} \sin(c_0 (\delta f - \delta f_i F) T_{FFT}) e^{j\psi_{ik} + n'_{ik}}
\]  
Eq 11

where the phase distortion due to synchronization errors is expressed by

\[
\psi_{ik} = \theta + 2\pi \delta t \left( kT + \frac{T_{FFT}}{2} + \delta t \right) + 2\pi \delta t \frac{i}{T_{FFT}}
\]

VI. ANALYTICAL EVALUATION OF THE BER

Analytical expressions for the BER are derived in this topic. Following [1], we start our analysis with defining the symbol transmitted as \( x_{k,n} \), which is an element of the symbol set \( \{ x_{k,n} \} \), \( m = \{ 1, 2, \ldots, M \} \). (Miss the order of the modulation scheme.) At the receiver’s site, an optimum detector will choose the symbol \( x_{k,n} \in \{ x_{k,m} \} \), which minimizes the distance metric

\[ M_d(x_{k,n}) = |y_k - \bar{x}_{k,n}|^2 \]  
Eq 13

This symbol is assumed most likely to be the transmitted symbol. The term \( X_k = h \cdot x \) in this equation accounts for the channel estimation. An error occurs when the metric calculated for a symbol \( x_{k,n} \not= x_{k,i} \) is smaller than the metric for the transmitted symbol \( x_{k,i} \). The probability of this event is written as

\[ P_e = P_r \{ M_d(x_{k,n}) < M_d(x_{k,i}) \} = P_r < 0 \]  
Eq 14

Where \( D = M_d(x_{k,n}) - M_d(x_{k,i}) \) is called the decision variable.

Assigning different constellation values to the variable \( x_{k,n} \not= x_{k,i} \), the probability can be calculated that an erroneous symbol \( x_{k,n} \) has been detected while the symbol \( x_{k,i} \) was transmitted. This allows, for many modulation schemes, an exact calculation of the BER.
for others, it allows the evaluation of close approximations

A. BPSK and QPSK

Exact results can be obtained for BPSK and QPSK modulation. The signal constellations for these techniques are depicted in Figure 5. For both schemes it is sufficient to consider (any) one transmitted symbol, due to symmetries. This symbol will be the +1, taken from the set \( \{x_{k,m}\} = \{1, -1\} \) for BPSK, and from \( \{x_{k,m}\} = \{1, j, -1, -j\} \) for QPSK. Note that \( |x_{k,m}|^2 = 1 \) for both modulation types.

![Figure 5 Selection of \( x_{k,i} \) and \( x_{k,n} \) for the performance evaluation of BPSK and QPSK.](image)

B. 8-PSK

Upper and lower bounds on the BER can be calculated for 8-PSK. An exact calculation is not possible because the eight signal states are not separable in the two orthogonal dimensions of the I/Q-plane.

C. 16-QAM

16-QAM can be evaluated without any error. This involves considering 4 different transmitted symbols occurring with equal probabilities and 24 error events. Some of them must be subtracted in order to account for overlapping decision regions.

![Figure 6 Error regions for 8-PSK when \( x_{k} = 1 \) was transmitted: (a) signal constellations and correct number of errors for each decision range, and (b) approximation by evaluating error probabilities from the three error states \( x_{k,n} \) shown.](image)

VII. RESULTS

Some observations can be made from the mathematical expressions derived

1. For coherent detection, the statistical parameters, and thus the performance results, only depend on \( P_0 \), \( \rho \), and \( \sigma N/2 \). In other words, the performance depends on the average SNR \( SNR \propto P_0 N / \sigma 2 \) and on the Rician K-factor \( K = \rho^2 / (P_0 - \rho^2) \).

2. The same holds in the limits \( F \to 0 \) or \( T \to 0 \) (i.e., for flat fading) for differential detection.

3. The performance of differential detection degrades for \( F > 0 \) (or \( T > 0 \)) because of a systematic estimation error in \( h_k = h_k - 1 + n_k \), because \( h_k - 1 \neq h_k \). The parameter products \( \tau_{rmsF} \) and \( \tau_{mT} \) define the degradation according to Performance results (average BER) for (1) and (2) and QPSK modulation are shown in Figure 5.13 as a function of the average SNR per bit (denoted \( E_b/N_0 \)) and as a function of \( K \).

![Figure 7 Performance of QPSK for coherent detection (perfect channel estimation) (“—”) and for differential detection with \( F = 0 \), that is, with perfect correlation between adjacent SCs (flat fading) (“—”).](image)

![Figure 8 Performance of different modulation schemes: (a) coherent detection with perfect channel estimation, and (b) differential detection with \( F = 0 \), that is, perfect correlation between adjacent SCs (flat fading).](image)
CONCLUSIONS

The derivation of the OFDM system model has confirmed that data symbols can be transmitted independently over multipath fading radio channels. It has to be assumed, however, that the channel’s maximum excess delay is shorter than the GI and that the system has been synchronized sufficiently. Small synchronization errors lead to systematic phase rotations of the data constellation points, a property that can be exploited for estimating synchronization offsets. If the timing- or frequency-synchronization error becomes too large, the orthogonality of the SCs is partly lost, and the SNR of the system is degraded; that is, ICI and ISI arise. ICI can also result from very fast channel variations (Doppler spreads) or from carrier phase jitters.

The system models presented can be utilized in analytical studies of various aspects of the OFDM technique, as, for instance, in the performance evaluation. The basic model introduced assumes perfect synchronization, while an extended model considers the phase rotations due to small synchronization offsets.

The performance analysis of an uncoded OFDM scheme is based on the classic formulas given by Proakis. Expressions are derived for the evaluation of different modulation schemes and for coherent and differential detection. The FD channel model for Rician fading channels has been applied. It allows performance results to be shown as a function of the channel parameters \( \{P_0, K, \tau_{\text{rms}}\} \), the NRP, the Rician K-factor, and the RDS.

REFERENCES


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