A DIFFERENT POLES PLACEMENT APPROACHES FOR PADE APPROXIMATION AND ROUTH APPROXIMATION

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Abstract—In this paper a different poles placement approach for pade approximation and routh approximation has been proposed. Pade approximation and routh approximation is used to convert higher order model into a lower order model. In this reduction a comparative analysis gives some new result about pade approximation and routh approximation for pole placement. For shifting the pole of higher order model, a novel approach has been proposed for the pole placement of reduced order model.

Keywords—Pade approximation, Routh approximation, Model order reduction, Reduced order model.

I. INTRODUCTION

In production industry it is often confronted with the analysis, design and synthesis of real-life problems. In development, the first step is to prepare a 'mathematical model' from a 'physical model', which can be considered as a substitute for the real problem [1]. The analysis and synthesis may not be economical due for the higher order model of the physical model. The gated higher order model has much more difficulty to analysis the problem and the desirable result expectation is also difficult for such complex system. So for better understanding and deep focus on parameters of the system, it is necessary to reduce the higher order model [2-4]. Thus it is necessary to obtain a lower order system to maintain the main characteristics of original system. The application of model order reduction in electrical engineering is a classical problem; that is due the fact that electrical engineering problems most often involve large scale system [5]. Pade approximation [6] and routh approximation [7] are the method of model order reduction for the higher order system. These methods give the simplification of a higher order model after converting it into a reduced order model. In general for a stable system, pade approximation method [8] produces the unstable system after reducing the order of original system; on the other hand routh approximation gives a stable reduced model for the same [9].

In this paper a new approach is proposed for the poles placement of reduced order model through pade approximation and routh approximation. This approach shows the repulsive or attractive behavior of poles for reduced order model due to shifting the pole position of original system.

II. STATEMENT OF PROBLEM

Pade approximation is one of the best methods of model order reduction as it gives the stable reduced order model. In general it gives an unstable reduced order model. On the other hand routh approximation always gives a stable reduced order model if original system is stable. So it is necessary to understand that, what is the effect of reduced order model poles possession after shifting the dominant pole of original system?

2.1. Higher order model transfer function

Let the original system [10] which is used as a plant in the control system can be represented by the transfer function expressed in equation (1) as

\[ G(s) = \frac{N(s)}{D(s)} = \frac{A_1 s^n + A_2 s^{n-1} + \ldots + A_r s^{n-r+1}}{A_1 s^n + A_2 s^{n-1} + \ldots + A_r s^{n-r+1}} \]  

2.2. Lower order model transfer function

To find a rth lower order for the above continuous system, where \( r < n \) in the following form, such that the lower order model retains all the characteristics of the original system and its response is as closely as possible to the original system response

\[ R(s) = \frac{a_{21} s^3 + a_{22} s^2 + \ldots + a_{2r} s^{r-1}}{a_{11} s^n + a_{12} s^{n-1} + \ldots + a_{1r} s^{n-r+1}} \]  

Where \( a_{2j} \) and \( a_{1j} \) are scalar constants and \( j=12,3,\ldots,n \).

Here, the objective is to study the pole location of reduced order model with shifting the dominant pole of reference model.

III. PADE APPROXIMATION

The rth-order reduced approximant \( R(s) \) for \( G(s) \) is obtained by pade approximation. This approach stems from the theory of pade and was later used for model reduction [11]. Before a formal presentation of the method is done, consider the following definition. Consider a function

\[ f(s) = c_0 + c_1 s + c_2 s^2 + \ldots \]  

For the function \( f(s) \) in Eq. 1 to be approximated, let the following Pade approximant be defined.
\[ \frac{U_n(s)}{V_n(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_{n-1} s^{n-1} + s^n} \]  
(4)

For the first \((m + n)\) terms of Eq. 3 and Eq. 4 to be equivalent, it becomes apparent that the following set of relations must hold:

\[ a_0 = b_1 c_0 \]

\[ a_i = b_i c_{i+1} + b_{i+1} c_i \]

\[ a_{n-1} = b_{n} c_{n-1} + b_{n-1} c_n \]

\[ 0 = b_{n} c_n + \ldots + b_1 c_0 \]

\[ 0 = b_0 c_{2n-1} + b_1 c_{2n-2} + \ldots + b_{n-2} c_n + c_{n-1} \]

Once the coefficients \(c_i (i = 0, 1, 2, \ldots, n)\) are found [3] using Eq. 5 then \(c_i = (-1)^i a_{n+i}\) gives the time moments of the original system.

The full model is

\[ G(s) = \frac{d_0 + d_1 s + d_2 s^2 + \ldots + d_{m} s^{m-1}}{e_0 + e_1 s + e_2 s^2 + \ldots + e_{m} s^{m-1}} \]

(6)

For the full model, describe in Eq. 6, Eq. 5 can be written in matrix form as,

\[
\begin{bmatrix}
   c_n & c_{n-1} & \ldots & c_1 \\
   c_{n+1} & c_n & \ldots & c_2 \\
   c_{n+2} & c_{n+1} & \ldots & c_3 \\
   \vdots & \vdots & \ddots & \vdots \\
   c_{2n-1} & c_{2n-2} & \ldots & c_n
\end{bmatrix}
\begin{bmatrix}
   b_0 \\
   b_1 \\
   \vdots \\
   b_{n-1}
\end{bmatrix}
= \begin{bmatrix}
   -c_0 \\
   -c_1 \\
   \vdots \\
   -c_{n-1}
\end{bmatrix}
\]

\[ \begin{bmatrix}
   c_0 & 0 & 0 & \ldots & 0 \\
   c_1 & c_0 & 0 & \ldots & 0 \\
   \vdots & \vdots & \ddots & \vdots & \vdots \\
   c_{n-2} & c_{n-1} & c_1 & c_0 & 0
\end{bmatrix}
\begin{bmatrix}
   b_0 \\
   b_1 \\
   \vdots \\
   b_{n-1}
\end{bmatrix}
= \begin{bmatrix}
   a_0 \\
   a_1 \\
   \vdots \\
   a_{n-1}
\end{bmatrix}
\]

(7)

IV. ROUHT APPROXIMATION

From Routh Approximation Algorithm [12]

\[ P_k(s) = \alpha_k P_{k-1}(s) + \beta_k \]

\[ Q_k(s) = \alpha_k Q_{k-1}(s) + Q_{k-2}(s) \]

Where \(k = 1, 2, 3, 4, \ldots \ldots \)

V. A NEW APPROACH ABOUT PADE APPROXIMATION & ROUHT APPROXIMATION

In this paper the discussion about the case when original system is stable and reduced 2\textsuperscript{nd} order model is stable in case of routh approximation but the same model have instability in case of pade approximation.

Let the full order model be,

\[ G(s) = \frac{d_0 + d_1 s + d_2 s^2 + \ldots + d_{m} s^{m-1}}{e_0 + e_1 s + e_2 s^2 + \ldots + e_{m} s^{m-1} + e_m s^m} \]

(10)

The system with pole zero form is expressed using Eq. (10) as,

\[ G(s) = \frac{(s - z_1)(s - z_2)(s - z_3)\ldots(s - z_n)}{(s - p_1)(s - p_2)(s - p_3)\ldots(s - p_m)} \]

(11)

Here, no. of poles > no. of zeros

Let the most dominant pole in the full order model be \(s = p_1\)

Let the second order reduced model be

\[ G'(s) = \frac{(s - z_1)}{(s + p_1)} \]

(12)

This indicates that the second order reduced model may be unstable and one pole may lie in the left half while other might be in the right half of the imaginary axis in the s-plane.

Here for routh approximation method when most dominant pole of original system is shifted towards the origin from the left half s-plane, the most dominant pole of reduced order model also shifted towards the origin. On the other hand in case of pade approximation when most dominant pole of original system is shifted towards the origin then the pole of reduced model show a repulsive and attractive behavior.

In case of routh approximation when the pole of original system shifted towards the origin in left half s-plane either it is dominant or non dominant, the poles of reduced order model also shifted towards the origin. It means the reduced order model poles have attractive nature about the origin.

On the other hand in case of pade approximation; if pole is shifted from \(-\alpha \to -0.5 \to -0.4\) then both pole of reduced model have repulsive nature about the origin but if most dominant pole shifted from \(-0.5 \to 0.0\) then reduced model poles have attractive nature about the origin and due to these formation, system gets unstable. Here \(-\alpha < -0.5\)

VI. EXAMPLES

Let we have a physical model which system transfer function after modeling is given as [10]

\[ G_s(s) = \frac{2 + 6s + 8s^2}{2 + 5s + 4s^2 + s^3} \]

(13)

Poles of the system are -2,-1,-1 indicates that the system is stable.
**TABLE: FOR ANALYSIS OF REDUCED ORDER MODEL POLES DUE TO, ORIGINAL SYSTEM POLE PLACEMENT**

<table>
<thead>
<tr>
<th>S.NO.</th>
<th>Original system</th>
<th>Reduced order model</th>
<th>Routh approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st dominant pole</td>
<td>2nd dominant pole</td>
<td>(10%) change</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>2</td>
<td>-1.8</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>3</td>
<td>-1.6</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>4</td>
<td>-1.4</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>5</td>
<td>-1.2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>-1</td>
<td>No change</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S.NO.</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st dominant pole</td>
<td>2nd dominant pole</td>
<td>(10%) change</td>
</tr>
<tr>
<td>7</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>8</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>9</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>10</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>11</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>12</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>13</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>14</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
<tr>
<td>15</td>
<td>-2</td>
<td>-1</td>
<td>No change</td>
</tr>
</tbody>
</table>

The salient points of the above study areas,

1. In the table it is shown that in all cases of pole placement either it is dominant or non dominant, for reduced order model in routh approximation follow the shifting of original system pole placement.
2. In the first part of table (S.NO. 1 to 6) it is shown that, if the non dominant pole of original system is shifted (10% decrease in each step) towards the origin then the poles of reduced order model shifted far away from the origin. It means pole have a repulsive nature about the origin.
3. (i) In the second part of the table (S.NO. 7 to 15) when the most dominant pole of original system is shifted (10 % decrease in each step) towards the origin from \(-x\) to \((-0.5 \text{ to } -0.4)\), then the pole of reduced order model system show the repulsive behavior about the origin.
4. (i) On the other hand if the both dominant pole of the original system shifted (10 % decrease in each step) towards the origin from \((-0.5 \text{ to } -0.4)\) to \(0\), then the pole of reduced order model system show the attractive behavior about the origin.

**CONCLUSIONS**

For the above study it can be concluded that the analysis of the shifting poles of original system and reduced order model gives some important result about the pade approximation and routh approximation. More than six examples have been tried out, the result is same as discussed here for given two examples. In the proposed approach, routh approximation gives the stable reduced order model if original system is stable. In pade approximation, it gives the unstable reduced order model due to the shifting dominant pole of original system from \(-x\) to \((-0.5 \text{ to } -0.4)\) and if stability lies in reduced order model then this is due to dominant pole of original system shifted from \((-0.5 \text{ to } -0.4)\) to \(0\). The pole of...
reduced order model behaves sometimes repulsive and some time attractive nature about the origin. For the same examples two different methods with same strategy generate the antonym result. All these happen due to the time moments generated in the intermediate stages of the formulation.

REFERENCES


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