International Journal Of Electrical, Electronics And Data Communication, ISSN: 2320-2084 Volume-3, Issue-12, Dec.-2015

VERIFICATION OF NETWORK THEOREMS FOR LINEAR ELECTRICAL CIRCUITS WITH LOSSY PASSIVE ELEMENTS USING FRACTIONAL-ORDER MODELING

[1]THALKAR HEMANGI ASHOK, [2]MUKESH D. PATIL, [3]VISHWESH A. VYAWAHARE

Department of Electronics Engineering, Ramrao Adik Institute of Technology,
Nerul, Navi Mumbai, India 400706
Department of Electronics and Telecommunication Engineering, Ramrao Adik Institute of Technology,
Nerul, Navi Mumbai, India 400706
Department of Electronics Engineering, Ramrao Adik Institute of Technology, Nerul, Navi Mumbai, India 400706
E-mail: [1]hemangi899@yahoo.com, [2]mukesh.rait@gmail.com, [3]vishwesh.vyawahare@rait.ac.in

Abstract — In this paper Network analysis is very important while doing any network related parameter operation which may include circuit design, simplification. But, there are many circuits can not be analyzed or simplified by any conventional circuit simplification method such as Kirchhoff's Voltage Law and Kirchhoff's Current Law, so we need to use some techniques which are very much important for simplification which further help to find actual output or response of circuit. For that we have Network Theorem using it circuit simplification gets very much faster so analysis becomes faster instead of getting we do not get actual output or response of circuit, which contains some losses due to circuit elements that circuit elements may be inductors and capacitors. For that we have to study about lossy components and their losses such as eddy current loss, iron loss and core loss. All the inductors and capacitors have losses. If we wish to consider this losses, then the resulting model is very bulky. So fractional order model of lossy inductor and for capacitor has been proposed. This project is an attempt to contribute part in linear network circuits containing lossy passive elements. Using this fractional order we are verifying our network theorem. Considering lossy elements we are going to implement the fractional equation for each of circuit elements which make circuit reliable which helps to reduce its disadvantages and contribute to many bigger circuits to reduce their losses. This project will helps many circuit designing and contributing to society as in many improved circuit performance.

I. INTRODUCTION

Many electric circuits are complex to analyse. To reduce their complexity for analysis, network theorems are used. These theorems in many cases provide insight into the circuit's operation that can not be obtained from mesh or nodal analysis. By using these theorems, the complicated circuits can be modelled with a simple equivalent network. Thus, the task of analysis gets reduced and simplified [4]. For the anal-ysis of electrical circuits, various theorems like Superposition theorem, Thevenin's theorem, Norton's theorem, etc are used. In these electrical circuits, the circuit elements are generally resistors, capacitors and inductors. The network theorems are applied assuming that the circuit elements have ideal characteristics. But practically it is observed that for the circuit elements like capacitor have dielectric loss where as inductor have core loss. The fractional-order models of these lossy elements (inductors and capacitors) gives better representation as compared to integer-order models. The fractional-order model of lossy inductor are proposed by Kruger and Schaffer and FO model of lossy capacitor loss proposed by Wester lund[10].

In this work, the network theorems are used for the analysis of electrical networks with lossy inductors and capacitors.

A. Introduction to Fractional Calculus:

All of us are familiar with normal derivatives integrals, like, $\frac{df(t)}{dt}$, $\frac{df^2(t)}{dt^2}$;we have first order ; second order derivative or first integral, double integral of a function. But everything in this world is including systems; Electrical networks, Graphs, Maps, Images. The traditional integral and derivative are, to say the least, a staple for the technology professional, essential as means of understanding and working with natural and artificial systems. So now we wish to have half-order, pi^{th} order or $(3\ 6i)^{th}$ order derivative or integral of a function. Fractional calculus means derivatives and integrals of arbitrary real or complex order. Born in 1695, it is as old as the conventional integral calculus, but the real application of this branch of mathematics came in the recent years with the rapid advances in sciences and the need of more complex, powerful and reliable systems. It was a result of useful contri-butions from mathematician like Leibniz(1695), Euler(1730), Lagrange(1772), Laplace(1812), Fourier(1822), Abel(1823), Liouville (1832)and Riemann(1876) who enriched this field with their invaluable efforts. Schaffer and Kruger for lossy inductor and Westerlund for lossy capacitor have provided the different equation techniques.[7][8][10]

1) Modeling of Lossy Coils using Fractional Derivatives:

Coil exposed to eddy current and hysteresis losses are conven-tionally described by an inductance with equivalent core-loss resistance connected in parallel. The value of the equivalent core-loss resistance depends on the working frequency and the external wiring. Thus the model is not satisfactory. The authors Schaffer and Kruger proposed lossy inductance using fractional derivatives containing both a loss term and a storage term. After introducing the theory of fractional derivatives, the operating mode of the fractional coil model is explained by the example of an RLC oscillating circuit. Subsequent measurements of a series resonant circuit with a lossy coil impressively confirm the theoretical model with regard to both the frequency and time domains. Usually, real coil shows a response characteristics that differs from that of an ideal inductance. Due to the inherent eddy current and hysteresis losses, this holds particularly true for coils with a conductive ferromagnetic core. The modeling of such coils is a challenge that has not been satisfactorily copied with until today. In the following, the behavior of a coil with a solid iron core based on the associated RLC series resonant circuit is shown as an examples[4] and a highly proficient and generally valid model to describe this coil is presented. The model is based on a non-integer rational derivative. The model's advantages are its closeness to reality, its simplicity and small number of parameters, although the technical term and theory of fractional derivatives seem unusual at first. However, they are a very helpful mathematical tool. Conventional type of equation for voltage across ideal inductor is given by

$$v_L = L * \frac{di(t)}{dt}.$$

But for lossy inductor, fractional-order model of inductor was proposed and validated by Schaffer and Kruger, The voltage-current relationship is given by [6],

$$V_{L\beta} = L_\beta * \frac{d^\beta i(t)}{dt^\beta}. \qquad (1)$$

Parameter β and L_β are the measures of core loss in the inductor.

2) Fractional-order Model of Inductor in s domain:: Take Laplace transform of (1) we get,

$$V_{L\beta}(s) = L_\beta * s^\beta I_L(s)$$

$$\frac{V_L(s)}{I_L(s)} = L_\beta s^\beta$$

Put $s = j\omega$

$$\frac{V_L(s)}{I_L(s)} = L_\beta * (j\omega)^\beta$$

$$\frac{V_L(s)}{I_L(s)} = j^\beta * (L_\beta * \omega^\beta) \qquad (2)$$

$$\frac{V_L(s)}{I_L(s)} = j^\beta * X_{L\beta}$$

$$\frac{V_L(s)}{I_L(s)} = (X_{L\beta}, \beta) = X_{L\beta}\left[\cos\frac{\beta\pi}{2} + j\sin\frac{\beta\pi}{2}\right] \qquad (3)$$

where

$$j^\beta = \left[\cos\frac{\beta\pi}{2} + j\sin\frac{\beta\pi}{2}\right]$$

B. Modeling of Lossy Capacitor using Fractional Derivative

The theory of capacitors and dielectrics is a difficult subject. Since, as it appears, almost every property that a dielectric possess does require its own theory. The voltage current relationship with ideal capacitor is given by

$$v_C = \frac{1}{C_0} * \int i(t) * dt$$

But for lossy capacitors, fractional order model of capacitor has been proposed and validated by Wester Lund [6], The voltage-current relationship is given by

$$V_C = \frac{1}{C_0} * J^\alpha * i(t)dt$$

1) Fractional-order Model of Capacitor in s domain.: As we know, for lossy capacitor, the voltage-current relationship is

$$V_C = \frac{1}{C_0} * J^\alpha * i(t) * dt \qquad (4)$$

Take Laplace transform of (4)

$$\frac{V_C(s)}{I_C(s)} = \frac{1}{C_\alpha * s^{-\alpha}}$$

Put $s = j\omega$

$$\frac{V_C(s)}{I_C(s)} = j^{-\alpha} * (C_\alpha * \omega^{-\alpha}) \qquad (5)$$

$$\frac{V_C(s)}{I_C(s)} = j^{-\alpha} * X_{C\alpha}$$

$$\frac{V_C(s)}{I_C(s)} = (X_{C\alpha}, \alpha) = X_{C\alpha}\left[\cos\frac{\alpha\pi}{2} - j\sin\frac{\alpha\pi}{2}\right] \qquad (6)$$

where

$$j^{-\alpha} = \left[\cos\frac{\alpha\pi}{2} - j\sin\frac{\alpha\pi}{2}\right]$$

II. INTRODUCTION TO NETWORK THEOREM

Many electrical circuits are complex, to reduce their complexity, to analyze them easily network theorems are used. These techniques in many cases do provide insight into the circuit's operation that can not be obtained from mesh or nodal analysis. We can model complicated circuit with a simple equivalent network, then the task of analysis gets greatly reduced and simplified. Some of the network theorems are as follows:

- Superposition Theorem
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem
- Millman's Theorem
- Reciprocity Theorem

A. *Superposition Theorem*

1) Replacing all other independent voltage sources with a short circuit (thereby eliminating difference of potential i.e. V = 0; internal impedance of ideal voltage source is zero (short circuit)).

2) Replacing all other independent current sources with an open circuit (thereby eliminating current i.e. I = 0; internal impedance of ideal current source is infinite (open circuit)).

In this work, the superposition theorem is applied to the electrical circuits with lossy inductors and capacitors. To the best of our knowledge this is an first attempt too analyze electrical circuits with lossy inductors and capacitors. The superposition theorem is applied in the examples given below for the circuits with lossy electrical elements.

Example 1: Determine the voltage across $(4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha)$ impedance for the network shown in fig 1.

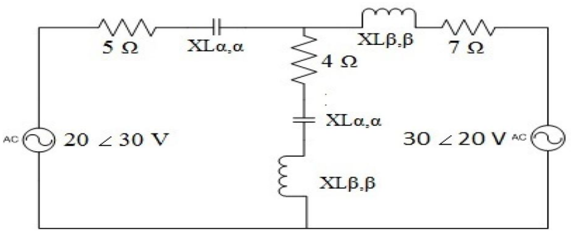

Fig. 1. Electrical circuit for example 1.

Calculating voltage across lossy inductor,
Consider $L_\beta = 0.09, \beta = 0.1, f = 50Hz$.
For the lossy inductor, the impedance is obtained by (2)

$$X_{L\beta} = (2 * \pi * f)^{\beta} * L_\beta, \qquad (7)$$

$$\begin{aligned} X_{L\beta} &= (2 * \pi * 50)^{0.1} * 0.09, \\ X_{L\beta} &= 0.16. \end{aligned}$$

The voltage current relationship;calculated β across inductor by (3),

$$(X_{L\beta}, \beta) = X_{L\beta}\left[\cos\left(\frac{\beta\pi}{2}\right) + j\sin\left(\frac{\beta\pi}{2}\right)\right], \qquad (8)$$

$$\begin{aligned} (X_{L\beta}, \beta) &= 0.16\left[0.999 + j(2.74 * 10^{-3})\right], \\ (X_{L\beta}, \beta) &= 0.1584 + j\left(4.384 * 10^{-4}\right). \end{aligned}$$

Calculating voltage across lossy capacitor,
Consider $C_\alpha = 0.07, \alpha = 0.3, f = 50Hz$.
The impedance across lossy capacitor is given by (5)

$$X_{C\alpha} = \frac{1}{(2 * \pi * f)^{\alpha} * C_\alpha}, \qquad (9)$$

$$\begin{aligned} X_{C\alpha} &= \frac{1}{(2 * \pi * 50)^{0.3} * 0.07}, \\ X_{C\alpha} &= 2.55. \end{aligned}$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent current is

$$\begin{aligned} I'_S &= (1.57 + j0.911) * \frac{7.1584 + j(4.384 * 10^{-4}}{13.87 - j0.0126)}, \\ I'_S &= 0.81 + j0.47A. \end{aligned}$$

By using Ohm's law; we get equivalent voltage,

$$V'_S = I'_S * (4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha). \qquad (14)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent voltage is

$$\begin{aligned} V'_S &= (0.81 + j0.47) * (6.71 - j0.017), \\ V'_S &= 5.44 + j3.15V. \end{aligned}$$

- STEP 2:When$30\angle 20V$ source is acting alone; short the voltage source $20\angle 30V$

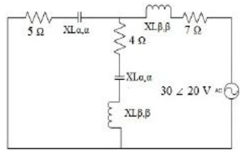

Fig. 3. Equivalent circuit with shorting First source for example 1.

The equivalent impedance of parallel impedances is given by,

$$Z_P = (5 + X_{C\alpha}, \alpha)||(4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha). \qquad (15)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent impedance is

$$\begin{aligned} Z_P &= \frac{7.55 - j0.021) * [6.71 - j0.017]}{7.55 - j0.021) + [6.71 - j0.017]}, \\ Z_P &= \frac{50.6601 - j0.2692}{14.26 - j0.038}, \\ Z_P &= 3.5526 - j(9.41 * 10^{-3})mho. \end{aligned}$$

By using Ohm's law; we get equivalent current,

$$I'' = \frac{30\angle 20}{Z_P + (7 + X_{L\beta}, \beta)}. \qquad (16)$$

Replacing $X_{L\beta}, \beta$ obtained in (7 and 8),the equivalent current is

$$\begin{aligned} I'' &= \frac{28.19 + j10.26}{[3.5526 - j(9.41 * 10^{-3}) + [7.16 + j(4.384 * 10^{-}}, \\ I'' &= \frac{28.19 + j10.26}{10.7126 - j(5.02 * 10^{-3})}, \\ I'' &= 2.63 + j0.96A. \end{aligned}$$

The voltage current relationship; calculated α across inductor by (6),

$$(X_{C\alpha}, \alpha) = X_{C\alpha}\left[\cos(\frac{\alpha\pi}{2}) - j\sin(\frac{\alpha\pi}{2})\right], \qquad (10)$$

$$\begin{aligned} (X_{C\alpha}, \alpha) &= 2.55\left[0.999 - j(8.22 * 10^{-3})\right], \\ (X_{C\alpha}, \alpha) &= 2.55 - j0.021. \end{aligned}$$

- STEP 1:When $20\angle 30V$ source is acting alone; short the voltage source $30\angle 20V$

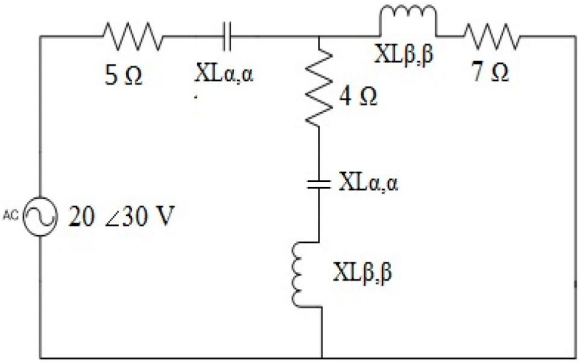

Fig. 2. Equivalent circuit with shorting second source for example 1.

The equivalent impedance of parallel impedances is given by,

$$Z_P = (7 + X_{L\beta}, \beta)||(4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha). \quad (11)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent impedance is

$$\begin{aligned} Z_P &= \frac{7.16 + j(4.384 * 10^{-3})) * [6.71 - j0.017]}{7.16 + j(4.384 * 10^{-3})) + [6.71 - j0.017]}, \\ Z_P &= \frac{48.0436 - j0.0923}{13.87 - j0.0126}, \\ Z_P &= 3.4638 - j(3.5 * 10^{-3})mho. \end{aligned}$$

By using Ohm's law; we get equivalent current,

$$I' = \frac{20\angle 30}{Z_P + (5 + X_{C\alpha}, \alpha)}. \quad (12)$$

Replacing $X_{C\alpha}, \alpha$ obtained in (9 and 10),the equivalent current is

$$\begin{aligned} I' &= \frac{17.32 + j10}{[3.4638 + j(3.5 * 10^{-3}) + [7.55 - j0.021]]}, \\ I' &= \frac{17.32 + j10}{11.0138 - j0.0245}, \\ I' &= 1.57 + j0.911A. \end{aligned}$$

By using Current divider rule;we get equivalent current,

$$I'_S = I' * \frac{7 + X_{L\beta}, \beta}{(4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha) + (7 + X_{L\beta}, \beta)}. \quad (13)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent current is

$$\begin{aligned} I'_S &= (1.57 + j0.911) * \frac{7.1584 + j(4.384 * 10^{-4}}{13.87 - j0.0126)}, \\ I'_S &= 0.81 + j0.47A. \end{aligned}$$

By using Ohm's law; we get equivalent voltage,

$$V'_S = I'_S * (4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha). \quad (14)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent voltage is

$$\begin{aligned} V'_S &= (0.81 + j0.47) * (6.71 - j0.017), \\ V'_S &= 5.44 + j3.15V. \end{aligned}$$

- STEP 2:When$30\angle 20V$ source is acting alone; short the voltage source $20\angle 30V$

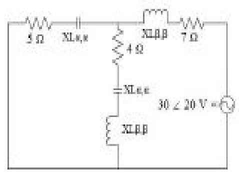

Fig. 3. Equivalent circuit with shorting First source for example 1.

The equivalent impedance of parallel impedances is given by,

$$Z_P = (5 + X_{C\alpha}, \alpha)||(4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha). \quad (15)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent impedance is

$$\begin{aligned} Z_P &= \frac{7.55 - j0.021) * [6.71 - j0.017]}{7.55 - j0.021) + [6.71 - j0.017]}, \\ Z_P &= \frac{50.6601 - j0.2692}{14.26 - j0.038}, \\ Z_P &= 3.5526 - j(9.41 * 10^{-3})mho. \end{aligned}$$

By using Ohm's law; we get equivalent current,

$$I'' = \frac{30\angle 20}{Z_P + (7 + X_{L\beta}, \beta)}. \quad (16)$$

Replacing $X_{L\beta}, \beta$ obtained in (7 and 8),the equivalent current is

$$\begin{aligned} I'' &= \frac{28.19 + j10.26}{[3.5526 - j(9.41 * 10^{-3}) + [7.16 + j(4.384 * 10} \\ I'' &= \frac{28.19 + j10.26}{10.7126 - j(5.02 * 10^{-3})}, \\ I'' &= 2.63 + j0.96A. \end{aligned}$$

By using Current divider rule;we get equivalent current,

$$I''_S = I'' * \frac{5 + X_{C\alpha}, \alpha}{(4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha) + (5 + X_{C\alpha}, \alpha}. \quad (17)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent current is

$$I''_S = (2.63 + j0.96) * \frac{7.55 - j0.021}{14.26 - j0.038},$$
$$I''_S = 1.41 + j0.51A.$$

By using Ohm's law; we get equivalent voltage,

$$V''_S = I''_S * (4 + X_{L\beta}, \beta + X_{C\alpha}, \alpha). \quad (18)$$

Replacing $X_{L\beta}, \beta$ and $X_{C\alpha}, \alpha$ obtained in (7 and 8) and (9 and 10) equivalent voltage is

$$V''_S = (1.41 + j0.51) * (6.71 - j0.017),$$
$$V''_S = 9.35 + j3.41V.$$

By adding V'_S and V''_S;we get total equivalent voltage,

$$V_S = V'_S + V''_S, \quad (19)$$

$$V_S = [5.44 + j3.15] + [9.35 + j3.41],$$
$$V_S = 14.79 + j6.54V.$$

TABLE I
RESULT OF SUPERPOSITION THEOREM BY VARYING β

Sr.No.	β	L_β	V'_S	V''_S	V_S
1.	0.1	0.09	$5.44 + j3.15$	$9.35 + j3.41$	$14.79 + j6.54$
2.	0.3	0.07	$5.55 + j3.22$	$9.27 + j3.34$	$14.82 + j6.56$
3.	0.5	0.05	$5.82 + j3.38$	$9.075 + j3.28$	$14.89 + j6.66$
4.	0.7	0.03	$6.20 + j3.61$	$8.82 + j3.18$	$15.02 + j6.79$
5.	0.9	0.01	$6.26 + j3.65$	$8.81 + j3.19$	$15.07 + j6.8$

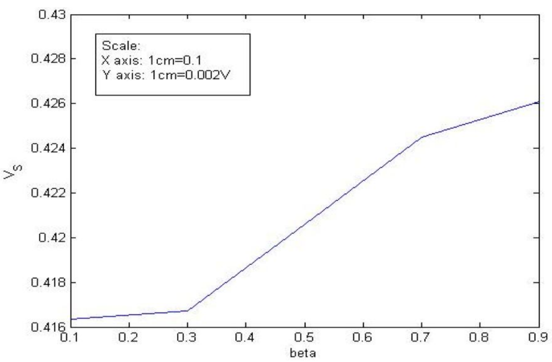

Fig. 4. Graph of superposition theorem by varying β for example 1.

RESULT: By solving above example using different values of β we get different superposition voltages;we conclude that β increases then superposition voltage is increases .

B. Thevenin's Theorem

- Any linear network with voltage and current sources and only impedance can be replaced at terminals A-B by an equivalent voltage source V_{th} in series connection with an equivalent impedance Z_{th}.
- This equivalent voltage Vth is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
- This equivalent impedance Z_{th} is the impedance obtained at terminals A-B of the network with all its independent current sources open circuited and all its independent voltage sources short circuited.

In this work, the Thevenin's theorem is applied to the electrical circuits with lossy inductors and capacitors. To the best of our knowledge this is an first attempt too analyze electrical circuits with lossy inductors and capacitors. The Thevenin's theorem is applied in the examples given below for the circuits with lossy electrical elements.

Example 2:
Using Thevenin's theorem,determine Z_{TH} and V_{TH} as shown in fig.5. below across A and B

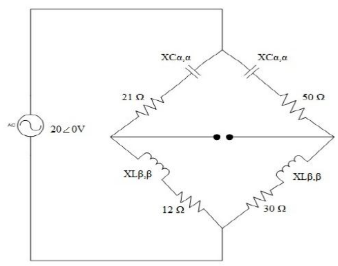

Fig. 5. Electrical circuit for example 2.

Calculating voltage across lossy capacitor,
Consider $C_\alpha = 0.09, \alpha = 0.1, f = 50Hz$.
The impedance across lossy capacitor is given by (5)

$$X_{C\alpha} = \frac{1}{(2 * \pi * f)^\alpha * C_\alpha}, \quad (20)$$

$$X_{C\alpha} = \frac{1}{(2 * \pi * 50)^{0.1} * 0.09},$$
$$X_{C\alpha} = 6.25.$$

The voltage current relationship;calculated α across inductor by (6),

$$(X_{C\alpha}, \alpha) = X_{C\alpha}\left[\cos(\frac{\alpha\pi}{2}) - j\sin(\frac{\alpha\pi}{2})\right], \quad (21)$$

$$(X_{C\alpha}, \alpha) = 6.25\left[0.999 - j(2.74 * 10^{-3})\right],$$
$$(X_{C\alpha}, \alpha) = 6.25 - j0.017.$$

Calculating voltage across lossy inductor,
Consider $L_\beta = 0.07, \beta = 0.3, f = 50Hz$.
For the lossy inductor, the impedance is obtained by (2)

$$X_{L\beta} = (2 * \pi * f)^{\beta} * L_\beta, \qquad (22)$$

$$\begin{aligned} X_{L\beta} &= (2 * \pi * 50)^{0.3} * 0.07, \\ X_{L\beta} &= 0.393. \end{aligned}$$

The voltage current relationship;calculated β across inductor by (3),

$$(X_{L\beta}, \beta) = X_{L\beta}\left[\cos(\frac{\beta\pi}{2}) + j\sin(\frac{\beta\pi}{2})\right], \qquad (23)$$

$$\begin{aligned} (X_{L\beta}, \beta) &= (X_{L\beta}, \beta) = 0.393\left[0.999 + j(8.225 * 10^{-3})\right], \\ (X_{L\beta}, \beta) &= 0.393 + j(3.23 * 10^{-3}). \end{aligned}$$

- STEP 1: The equivalent voltage obtained with terminals A-B open circuited.

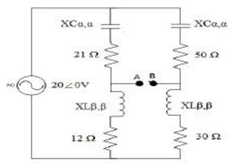

Fig. 6. Equivalent circuit with terminals A-B open circuited for example 2.

Apply KVL to first loop in fig.6.,the equivalent current is

$$20\angle 0 - [(12 + X_{L\beta,\beta} + (21 + X_{C\alpha,\alpha})] * I = 0, \qquad (24)$$

$$I = \frac{20}{[(12 + X_{L\beta,\beta} + (21 + X_{C\alpha,\alpha})]}.$$

Replacing $(X_{L\beta}, \beta)$ and $(X_{C\alpha,\alpha})$ obtained in (22 and 23) and (20 and 21),the equivalent current is

$$\begin{aligned} I &= \frac{20}{39.633 - j0.014}, \\ I &= 0.505 + j(1.75 * 10^{-4})A. \end{aligned}$$

Apply KVL to second loop in fig.6.,the equivalent current is

$$20\angle 0 - [(30 + X_{L\beta,\beta} + (50 + X_{C\alpha,\alpha})] * I = 0, \qquad (25)$$

$$I = \frac{20}{[(30 + X_{L\beta,\beta} + (50 + X_{C\alpha,\alpha})]}.$$

Replacing $(X_{L\beta}, \beta)$ and $(X_{C\alpha,\alpha})$ obtained in (22 and 23) and (20 and 21),the equivalent current is

$$\begin{aligned} I &= \frac{20}{86.633 - j0.014}, \\ I &= 0.231 + j(3.67 * 10^{-5})A. \end{aligned}$$

By using ohm's law,we get equivalent voltage

$$V_{TH} = V_A - V_B, \qquad (26)$$

$$V_{TH} = (12 + X_{L\beta,\beta}) * I_1 - (30 + X_{L\beta,\beta}) * I_2.$$

Replacing $(X_{L\beta}, \beta)$ obtained in (22 and 23),the equivalent current is

$$V_{TH} = -0.76 + j(1.95 * 10^{-3})V.$$

- STEP 2: Equivalent impedance obtained with voltage source short circuited.

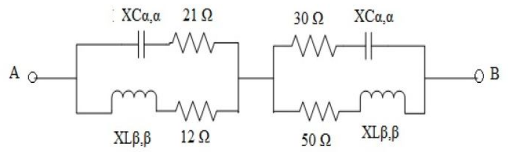

Fig. 7. Equivalent circuit with shorting voltage source for example 2.

The equivalent impedance of parallel impedances is given by,

$$Z_{TH} = (12 + X_{L\beta,\beta}||21 + X_{C\alpha,\alpha}) + (50 + X_{L\beta,\beta}||30 + X_{C\alpha} \qquad (27)$$

Replacing $(X_{L\beta}, \beta)$ and $(X_{C\alpha,\alpha})$ obtained in (22 and 23) and (20 and 21),the equivalent current is

$$\begin{aligned} Z_{TH} &= 8.52 - j1.44 * 10^{-4}) + (21.08 - j(5.31 * 10^{-} \\ Z_{TH} &= 29.60 - j(5.45 * 10^{-3})mho. \end{aligned}$$

TABLE II
RESULT OF THEVENIN'S THEOREM BY VARYING α

Sr.No.	α	C_α	V_{TH}
1.	0.1	0.09	$-0.76 + j(1.95 * 10^{-3})$
2.	0.3	0.07	$-0.35 + j(2.87 * 10^{-3})$
3.	0.5	0.05	$-0.28 + j(2.57 * 10^{-3})$
4.	0.7	0.03	$-0.22 + j(2.095 * 10^{-3})$
5.	0.9	0.01	$-0.198 + j(2.44 * 10^{-3})$

RESULT By: solving above example using different values of α we get different Thevenin's voltages;we conclude that α increases then Thevenin's voltage is decreases

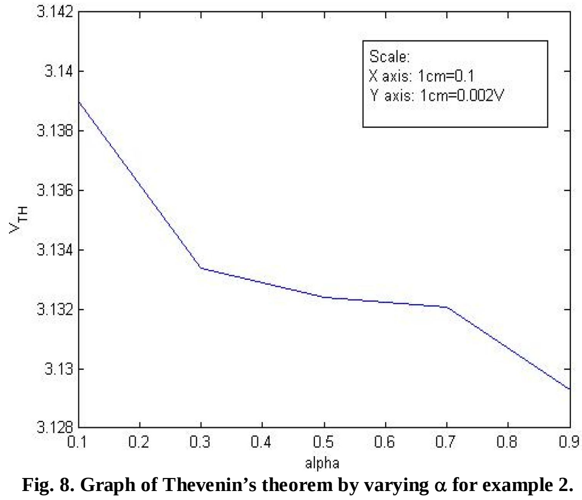

Fig. 8. Graph of Thevenin's theorem by varying α for example 2.

CONCLUSION

In the literature, it is proved that all inductors have core loss and capacitors have dielectric loss. These losses have impact on the response of the electrical circuits. In this work, the analysis of various network theorems is carried out. The results are presented for various values of α and β which describes the amount of dielectric loss and core loss in capacitors and inductors respectively. This is a first attempt to anlyse electrical theorem with lossy electrical elements.

ACKNOWLEDGMENT

I sincerely thank to the Management and Principal of Ramrao Adik Institute of Technology, Nerul, Navi Mumbai for providing me financial support and thorough cooperation to attend this conference.

REFERENCES

[1] K. S. Miller and B. Ross. An Introduction to the fractional calculus and fractional differential equations. John Wiley and Sons, USA, 1993.
[2] K. B. Oldham and J. Spanier. The Fractional Calculus. Dover Publications, USA, 2006.
[3] MATLAB Manual. Mathworks, USA, 2013a.
[4] ‘Electrical Circuits Theory”, Ravish sing,vol:05,Person Publication.
[5] A. G. Radwan and K. N. Salama. Passive and active elements using fractional $L_{\beta}C_{\alpha}$ circuit. IEEE Transactions on Circuits and Systems, 58(10):2388-2397, 2011.
[6] I. Schafer and Klaus Kruger. Modelling of coils using fractional derivatives. Journal of Magnetism and Magnetic Materials, 307:91-98, 2006.
[7] Ingo Schafer and Klaus Kruger. Modelling of lossy coils using fractional derivatives. Journal of physics, Applied physics, 41, 2008.
[8] Z. Garczarczyk. State equations for fractional RLC circuit. In XV International Symposium on Theoretical Engineering, Lubeck, Germany, June 2009.
[9] I. S. Jesus and J. A. Tenreiro Machado. Comparing integer and fractional models in some electrical systems. In The 4th IFAC Workshop on Fractional Differentiation and its Applications, Badajoz, Spain, October 2010.
[10] Westerlund and L. Ekstam. Capacitor theory. IEEE Transactions on Dielectrics and Electrical Insulation, 1(5):826-839, 1994.
[11] Wajdi Aehmad,Phd,Power factor correction using Fractional Capacitor, Electrical and Electronics,Electrical and Electronics Engineering Dept. University of Sharjah, P. 0. Box 27272 Sharjah, Emirate 2003
[12] P.L. Chapman S.D. Sudhoff, Dynamic Lossy Inductor Model for Power Converter Simulation,International journal of Circuit Theory and Application, 2002
[13] A. M. A. El-Sayed, H. M. Nour, W. E. Raslan, E. S. El-Shazly,Fractional Parallel RLC Circuit, Alexandria journal of Mathematics, vol.3, NO.1, June 2012.

★★★