BATT ALGORITHM BASED OPTIMAL TRANSMISSION EXPANSION PLANING WITH GENERATOR LOAD MODELS AND FREQUENCY CONTROLS

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Abstract—Basically, Static planning is standard TEP. Now-a-days dynamic planning also made on TEP for a selected period. TEP is first implemented as DC-TEP is approximate planning. In practical scenario AC-TEP is the best selection for TEP. Different metaheuristic techniques are used to solve transmission expansion planning (TEP) problem using an AC model. In this Paper TEP is carried out by using bat algorithm. Bat algorithm (BA) is a population based metaheuristic approach based on hunting behavior of bats. In this algorithm possible solution of the problem is represented by positions taken by bat in its approach towards its target. Quality of the solution is indicated by the best position of a bat to its prey. BA has been tested for continuous constrained optimization problems. BA can be used to solve many power system optimization problems. The objective of the TEP problem is to optimize the transmission network topology by selecting the new circuits that should be added to an existing transmission network so as to minimize the objective function. It is subjected to operating conditions for generating units and transmission network.

Keywords— AC-OPF, Bat Algorithm, Frequency Control, Generator Model, Load Models, Transmission Expansion Planing.

1. INTRODUCTION

Transmission Expansion Planning (TEP) addresses the problem of augmenting an existing transmission network to optimally serve a growing electric load while satisfying a set of economical, technical and reliability constraints. In general, TEP is a stochastic decision making problem that consists of determining the time, the location and the number of the transmission lines to be built. TEP can be classified from various points of view. Based on the solution methods there are three types of algorithms to solve the planning problem: 1) mathematical optimization methods, 2) heuristic methods and, 3) combinatorial methods called metaheuristic methods [17].

Transmission expansion planning can also be categorized as static and dynamic planning. In the dynamic expansion planning, the constructing time of lines will be determined in the optimization process, while in the static one there is only a “target year” in which the selected optimal lines should be built. The planners of the power system will face many uncertainties during the planning. The papers reviewed in the literature survey use a DC approach [9] in order to solve the TEP problem which is not completely suitable because of ignoring the reactive power. This paper proposes an approach for transmission planning based on AC optimal power flow (AC-OPF). Using AC-OPF could provide us a more precise picture of the active and reactive power flows in the expanded power network. Although the new model is more complicated than previous models, in the future, it certainly leads to more precise and optimal plan. In order to handle the various objectives a single objective optimization framework is presented. The bat algorithm exploits the so called echolocation of bats [10]. Bats use sonar echoes to detect and avoid obstacles. It is generally known, that sound pulses are transformed to frequency which reflects from obstacle. Bats can use time delay from emission to reaction and use it for navigation. They typically emit short loud, sound impulses. The pulse rate is usually defined as 10 to 20 times per second. After hitting and reflecting, bats transform their own pulse to useful information to gauge how far away the prey is. Bats are using wavelengths, that vary from range [0.7 - 17] mm or inbound frequencies [20,500] kHz. By implementation, pulse frequency and rate has to be defined. Pulse rate can be simply determined from range 0 to 1, where 0 means there is no emission and by 1, bats are emitting maximum.

II. FREQUENCY CONTROLS AND MODELS

A. Generator Models

Under normal conditions, system frequency is maintained constant and generators are operated at a scheduled voltage and output. When system load changes, however, output of generators is varied by instruction from the Automatic Load Frequency Control (AFC). Governor setting is changed so that operation returns to a point on the governor load-speed curve corresponding to system operating frequency. When the system is disturbed by loss of generation or tie line support, the governor restores balance automatically, while generator terminal voltage is kept to a reference voltage within the limits of the exciter rating, and some generators are
controlled automatically by their reactive power output.

Generator real power output is adjusted by the static response of the prime mover. This may be expressed as

\[ P_g = P_{go} - \frac{P}{R} \Delta f \]  

and

\[ P_{mn}^m \leq P_g \leq P_{mn}^m \]  

where

- \( P_g \) - is the real power output of the generator at \( i^\text{th} \) bus,
- \( P_{go} \) - is the scheduled power output of the generator at \( i^\text{th} \) bus,
- \( P \) - is the rated output of the generator,
- \( R \) - is the speed regulation in per unit.

**B. Frequency Controls**

**Automatic Load-frequency Control (AFC):** Frequency fluctuations due to changes in load are monitored continuously, and the frequency is maintained constant by using a governor motor or limit to control generator output. AFC does this automatically and are

- (i) Flat Frequency control (FFC),
- (ii) Flat Tie-line Control (FTC), and
- (iii) Flat Tie-line frequency Bias Control (TBC) on interconnected systems.

**Flat Frequency control (FFC):** In response to changes in system frequency, the power outputs of generators within a prescribed area are automatically regulated to maintain scheduled system frequency. Since the system frequency remains constant (i.e., \( \Delta f = 0 \)), \( \Delta f \) is replaced by the new variable \( P_{N0} \).

Power output for the regulating generators is

\[ P_g = P_{go} + \alpha_i P_{N0} \]  

Where

- \( P_g \) - is the real power output of the generator at \( i^\text{th} \) bus,
- \( P_{go} \) - is the scheduled power generation at \( i^\text{th} \) bus,
- \( P_{N0} \) - is the supply insufficiency in a given area,
- \( \alpha_i \) - is the load distribution factor of \( i^\text{th} \) generator such that \( \sum \alpha_i = 0 \).

**C. Load Models**

Load models are traditionally classified into two broad categories, static models and dynamic models.

**Static Load Models:** These models express the active and reactive powers, at any instant of time, as a function of the bus voltage magnitude and frequency. Static load models are used in both static and dynamic load components.

**Dynamic Load Model:** A Dynamic load model expresses the active and reactive powers at any instant of time as functions of the voltage magnitude and frequency. Studies of inter area oscillations, voltage stability, and long term stability often require load dynamic to be modeled.

**Different types of Static and Dynamic Load Models:**

The following different types of static and dynamic load models are considered:

- **a. Constant impedance load model** is a static load model where the power varies directly with the square of the voltage magnitude. It may also be called a constant admittance load model.

- **b. Constant current load model** is a static load model where the power varies directly with the voltage magnitude.

- **c. Constant power load model** is a static load model where the power does not vary with changes in voltage magnitude. It may also be called constant MVA load model.

The static load model that represents the power relationship to voltage magnitude as a polynomial equation, usually in the following form:

\[ P_a = P_{a0} (1 + C_a \Delta f) \left[ K_{pp} + K_{pq} \left( \frac{v}{v_0} \right) + K_{q,q} \left( \frac{v}{v_0} \right)^2 \right] \]  

\[ Q_a = Q_{a0} (1 + C_a \Delta f) \left[ K_{p,q} \left( \frac{v}{v_0} \right) + K_{q,q} \left( \frac{v}{v_0} \right)^2 \right] \]  

where

- \( P_a \) - active power demand at \( i^\text{th} \) bus,
- \( Q_a \) - reactive power demand at \( i^\text{th} \) bus,
- \( P_{a0} \) - active power consumptions at rated voltage, \( v_0 \) at \( i^\text{th} \) bus,
- \( Q_{a0} \) - reactive power consumptions at rated voltage, \( v_0 \) at \( i^\text{th} \) bus,
- \( C_a, C_q \) - constant of frequency characteristics of load,
- \( v \) - supply voltage,
- \( v_0 \) - rated voltage.

**III. TEP PROBLEM FORMULATION**

TEP problem is usually refers as a static transmission model. Generally, the objective of fitness function is to find optimal solution, measure performance of candidate solutions and check for violation of the planning problem constraints. Fitness function of the static TEP problem is basically a combination between objective function and penalty functions. The purpose of applying penalty functions to the fitness function is to represent violations of equality and inequality constraints. In this static TEP problem, there are two equality constraints, which is node balance of AC active and reactive power flow. In contrast, there are several inequality constraints to be considered, namely power flow limit on transmission lines constraint, active and reactive power generation limit, injection right of way constraint and bus voltage limit.

The TEP based on AC-OPF problem can be mathematically expressed as follows:
\[ \text{Min} \quad F(x,u) \]

Subject to \( b(x,u) = 0; \ w_{\text{min}} \leq w \leq w_{\text{max}} \)

Where: \( F \) – is an objective function which has to be minimized
\( X \) – is a static vector which denotes the dependent variables
\( V \) – is a vector representing all control variables
\( b \) – is equality constraint which is active and reactive power equilibrium condition
\( W \) – is inequality constraints which represents limits of control variables and system operating limits.

D. Objective Functions

**Minimization of investment cost:** The objective of this function is to minimize the investment cost \( \text{ICof} \) of the system and this objective can be formulated as follows:
\[ \text{IC} = \sum_{y=1}^{n_c} c_y n_y \]

Where \( \text{IC} \) is the investment cost, \( c_y \) is the cost of the candidate circuit for addition to the branch \( i-j \) and \( n_y \) is number of circuits added to the branch \( i-j \) and \( n_l \) is the number of candidate circuits.

**Minimization of operation cost:** The objective of this function is to minimize the operation cost \( \text{OC} \) of the system and this objective can be formulated as follows:
\[ \text{OC} = \sum_{i=1}^{n_l} (a_i P_{gi}^2 + b_i P_{gi} + c_i) \]

Where \( \text{OC} \) is the operation cost (fuel cost), \( P_{gi} \) is the power generation of \( i^{th} \) generator, \( a_i, b_i \) and \( c_i \) are the constant coefficients of power generation

**Minimization of real power loss:** The objective of this function is to minimize the real power loss \( PL \) of the system and this objective can be formulated as follows:
\[ PL = \sum_{y=1}^{n_c} G_y (n_y) (v_i^2 + v_j^2 - 2 v_i v_j \cos(\delta_i - \delta_j)) \]

Where \( PL \) is the real power loss of the system, \( n_y \) is the number of candidate lines between buses \( i \) and \( j \), \( v_i, v_j \) are the bus voltages and \( \delta_i \) and \( \delta_j \) are the voltage angles at buses \( I \) and \( j \) respectively.

**State Variables:** The state variables of a TEP based on AC-OPF consists of active power generation at slack bus \( (P_{gi}) \), voltage magnitude and phase angles of all load buses \( (v_i, \delta_i) \), generator reactive power outputs \( (Q_{gi}) \), transmission line loadings \( (S_{ij}) \). The state vector \( X \) can be expressed as:
\[ X = [P_{gi}, v_i, \ldots, v_n, Q_{gi}, \ldots, Q_{ng}, S_{ij}, \ldots, S_{nl}] \]

where \( n_l, n \) and \( n_g \) represents the number of transmission lines, number of buses and number of generators respectively.

**Control Variables:** The control variables of TEP based on AC-OPF consists of active power generation output \( (P_{gi}) \), number of additional lines \( (n_l) \) and reactive power injections \( (Q_{qi}) \). The control vector \( u \) can be represented as:
\[ u = [P_{g1}, \ldots, P_{gn}, n_1, \ldots, n_n, Q_{q1}, \ldots, Q_{qn}] \]

where \( n_g, n_l \) and \( n_q \) are the number of generators, number of transmission lines and number of reactive power injections

E. Constraints

The TEP based on AC-OPF has to follow equality and inequality constraints strictly. The real and reactive power equilibrium is considered as equality constraints and various operating limits are considered as inequality constraints.

**Equality constraints:** These constraints are shown load flow equations of both active power and reactive power.
Mathematically represented as follows:
\[ V_j \sum_{j=1}^{n} V_j (G_{ji}(n) \cos(\delta_i - \delta_j) + B_{ji}(n) \sin(\delta_i - \delta_j)) - P_{gi} + P_{qj} = 0 \]

\[ V_j \sum_{j=1}^{n} V_j (G_{ji}(n) \sin(\delta_i - \delta_j) + B_{ji}(n) \cos(\delta_i - \delta_j)) - Q_{ji} + Q_{qi} = 0 \]

where \( P_{gi}, Q_{qi} \) are active and reactive power generations at \( i^{th} \) bus respectively. \( P_{qj}, Q_{qj} \) are active and reactive power demands at \( j^{th} \) bus respectively. \( q_j \) is injected reactive power at \( i^{th} \) bus.

\( n \) is number of added lines vector. \( G \) and \( B \) are the conductance and susceptance matrices respectively and are given by
\[ G = \begin{bmatrix} G_{ij}(n) - (n_{ij} g_{ij} + n_{ij}^0 g_{ij}^0) \\ G_{ij}(n) = \sum_{j=1}^{n} (n_{ij} g_{ij} + n_{ij}^0 g_{ij}^0) \end{bmatrix} \]

\[ B = \begin{bmatrix} B_{ij}(n) - (n_{ij} b_{ij} + n_{ij}^0 b_{ij}^0) \\ B_{ij}(n) = \sum_{j=1}^{n} (n_{ij} b_{ij} + n_{ij}^0 b_{ij}^0) \end{bmatrix} \]

where \( g_{ij}^0 \) and \( b_{ij}^0 \) are conductance and susceptance of the transmission line connected between buses \( i \) and \( j \) at initial conditions and \( g_{ij} \) and \( b_{ij} \) are conductance and susceptance of the transmission line connected between buses \( i \) and \( j \) at added line condition. \( n_{ij}^0 \) and \( n_{ij} \) are the number of lines between buses \( i \) and \( j \) at initial and added line condition respectively.
Inequality constraints: These constraints represent the system operating limits and are as follows:

Generation constraints: Generator voltages, real power outputs and reactive power outputs should be within their minimum and maximum limits.

\[
\frac{v_{i \text{min}}}{\text{min}} \leq v_i \leq \frac{v_{i \text{max}}}{\text{max}}, \quad gl = 1, ..., ng
\]  
(16)

\[
P_{i \text{min}} \leq P_i \leq P_{i \text{max}}, \quad gl = 1, ..., ng
\]  
(17)

\[
Q_{i \text{min}} \leq Q_i \leq Q_{i \text{max}}, \quad gl = 1, ..., ng
\]  
(18)

Reactive power injection limits: Reactive power injection at load buses should be within minimum and maximum limits

\[
q_{i \text{min}} \leq q_i \leq q_{i \text{max}}, \quad ci = 1, ..., nq
\]  
(19)

Security limits: These constraints include the limits of voltage magnitudes at all load buses, voltage angles at all buses and transmission line loadings of candidate lines.

\[
v_{i \text{min}} \leq v_i \leq v_{i \text{max}}, \quad i = 1, ..., npq
\]  
(20)

\[
\delta_{i \text{min}} \leq \delta_i \leq \delta_{i \text{max}}, \quad i = 1, ..., nb
\]  
(21)

\[
(n + n^h)_{i \text{min}} \leq (n + n^h)_{i} \leq (n + n^h)_{i \text{max}}, \quad l = 1, ..., nl
\]  
(22)

Number of lines limits: The number of lines added in the candidate lines should not exceed the maximum number of lines.

\[
0 \leq n_i \leq n_{i \text{max}}, \quad i = 1, ..., nl
\]  
(23)

IV. BAT ALGORITHM FOR TEP PROBLEM

The proposed optimization program is expected to be able to solve a number of mathematical and engineering problems, such as economic power dispatch, unit commitment, optimal power flow, power system planning, transmission expansion planning, etc. The overall procedure of the BA optimization program for TEP based on AC-OPF has been described as follows:

Step 1: Set up all required parameters of the BA optimization process i.e. control parameters of the BA optimization process that are population size (NP), loudness (A), pulse rate (r), velocities (v), termination criterion (G), number of problem variables (D), lower and upper bounds of frequency (f_{i \text{min}} and f_{i \text{max}}), lower and upper bounds of initial population (x_{i \text{min}} and x_{i \text{max}}) and maximum number of iterations or generations (G_{\text{max}});

Step 2: Set generation G = 0 for initialization step of BA optimization process;

Step 3: Initialization step i.e. Initialize population P of individuals according to equation (4.19) where each decision parameter in every vector of the initial population is assigned a randomly selected value from within its corresponding feasible bounds;

\[
x_i = x_{i \text{min}} + \text{rand} \left( x_{i \text{max}} - x_{i \text{min}} \right)
\]  
(24)

Step 4: For each generation check the equality criteria i.e. sum of all generations should be equal to load demand

\[
\sum_{i=1}^{NG} P_i = P_d
\]  
(25)

Step 5: Run Optimum power flow

Step 6: Calculate and evaluate the fitness values of the initial individuals according to the problem’s fitness function;

Step 7: Rank the initial individuals according to their fitness;

Step 8: Set iteration G = 1 for optimization step of BA optimization process;

Step 9: Generate new solutions by adjusting frequency and updating velocities and locations based on equations (26-28)

\[
f_i = f_{i \text{min}} + \text{rand} \left( f_{i \text{max}} - f_{i \text{min}} \right)
\]  
(26)

\[
v_i = v_{i-1} + (x_{i}^* - x_{i}) f_i
\]  
(27)

\[
x_i = x_{i-1}^* - v_i
\]  
(28)

x_i is the current best global solution

Step 10: if rand > r_i select a solution among the best solutions

Step 11: Generate a local solution around the best solution,

Step 12: End if

Step 13: For each generation check the equality criteria i.e. sum of all generations should be equal to load demand

\[
\sum_{i=1}^{NG} P_i = P_d
\]  
(29)

Step 14: Run optimum power flow

Step 15: Evaluate fitness and objective function for the best solution

Step 16: if (rand < A & f(x) < f(x_i)) accept the new solution

Step 17: Increase r_i and reduce A

Step 18: End if

Step 19: Rank the bats and find the current best X

Step 20: Check the termination criteria; i.e. the number of current generation remains not over the maximum number of generations \( G < G_{\text{max}} \), set \( G = G + 1 \) and return to step 9 for repeating to search the solution. Otherwise, stop to calculate and go to step 21;

Step 21: Post-processing the results

V. SIMULATION RESULTS

The Transmission Expansion Planning is executed using Bat Algorithm has been implemented in MATLAB 7.10.0.499, 32 bit, 3 GB RAM, Intel Core
2 Duo T6600 2.20 GHz Processor with Windows 7 Operating System. Garver 6 – bus standard electrical transmission network is considered for the expansion planning. The TEP problem has been investigated in three case studies. The objective functions are investment cost, operation cost and power losses are tested in case studies. The generated power at each generator varies between $P_{gmin}^i$ and $P_{gmax}^i$.

Garver 6 – bus system consists of 6 buses, 15 possible branches, 760 MW, 152 MVAR demand and maximum 5 lines can be added to each branch. The complete date of the system is available in [3] and also shown in Fig.1. The dotted lines represent new possible line additions and solid lines are the existing lines. The detailed data of the test system was available in the above paper. Reactive power injections are at all the load buses 2, 4 and 5, the upper and lower limit of bus voltages are 1.05 and 0.95 p.u respectively are considered. The minimum generation $g_{min}^i$ of each generator is considered as zeros, the maximum generation $g_{max}^i$ of each generator resistance and reactance of candidate line, permissible line loading of each line is given in [14].

To obtain optimum values of all objective functions, the algorithm was run for 10 times. BA parameter are $D=15$, $NP=50$, $A=0.9855$, $r=0.0125$ and maximum iterations are 500.

TEP has been studied for Garver – 6 bus system by considering three (3) case studies are as follows:

Case – I: Reactive power is not injected at any load bus, investment cost $130 million obtained in base case [12].

Case – II: The selection of reactive power injections at all load buses has been varied between minimum and maximum limits (0 MVAR and 0.1 MVAR respectively). Thus the investment cost $110 million obtained in base case [13].

Case – III: The selection of reactive power injections at all load buses has been varied between minimum and maximum limits (0 MVAR and 1.5 MVAR respectively) so that investment cost has been decreased to $ 80 million compared to Case – I and Case – II.

F. Case- I results

Transmission Expansion Planning based on AC-OPF using AC network. TEP using DEA with various DE parameters and system parameter and constraints are considered. In this case reactive power is not injected at any load buses. The results are tabulated are shown in TABLE 1 for investment cost, operation cost and real power loss as the objective function simultaneously.

Without injection of reactive power at load buses, investment cost as objective function, investment cost $130 million was in base case and the optimum investment cost $80 million was resulted in combination of flat frequency control generator model and constant impedance load model i.e. FFC+CZ LM.

Table 1: Summarized Results of Garver 6-Bus System for Case – I

<table>
<thead>
<tr>
<th>Model/Objective Function</th>
<th>Investment Cost ($x10^6)</th>
<th>Operation Cost ($/h)</th>
<th>Power Loss (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>130</td>
<td>614,514</td>
<td>2.9729</td>
</tr>
<tr>
<td>CP LM</td>
<td>130</td>
<td>614,5018</td>
<td>2.9729</td>
</tr>
<tr>
<td>CC LM</td>
<td>100</td>
<td>606,7455</td>
<td>2.9316</td>
</tr>
<tr>
<td>CZ LM</td>
<td>110</td>
<td>595,2889</td>
<td>2.8936</td>
</tr>
<tr>
<td>Mixed LM</td>
<td>110</td>
<td>612,2883</td>
<td>2.9506</td>
</tr>
<tr>
<td>FFC</td>
<td>130</td>
<td>614,512</td>
<td>2.9825</td>
</tr>
<tr>
<td>FFC + CP LM</td>
<td>130</td>
<td>614,5048</td>
<td>2.9825</td>
</tr>
<tr>
<td>FFC + CC LM</td>
<td>100</td>
<td>606,6354</td>
<td>2.9149</td>
</tr>
<tr>
<td>FFC + CZ LM</td>
<td>80</td>
<td>595,0061</td>
<td>2.8507</td>
</tr>
<tr>
<td>FFC + Mixed LM</td>
<td>100</td>
<td>612,2823</td>
<td>2.9486</td>
</tr>
</tbody>
</table>

G. Case- II results

The reactive power injection at all load buses was varied between upper and lower limits (0 MVAR and 0.1 MVAR respectively). The optimum values of investment cost, operation cost and power loss as objective function are presented in TABLE 2.
With injection of reactive power at all load buses, investment cost $110 million was in base case where as the optimum investment cost $80 million resulted in (a) constant impedance load model (CZ LM), (b) combination of flat frequency control generator model and constant current load model (FFC+CC LM) and (c) combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

By maintaining the system security and all constraint limits, operation cost as the objective function, optimum operation cost 594.9293 $/h resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM). The healthy system voltage profile has been maintained at all buses and also line loadings are within the permissible limits in all lines. The healthy system voltage profile has been maintained at all buses and also line loadings are within the permissible limits in all lines. Power loss as the objective function, the optimum real power loss 2.818 MW resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

H. Case-III results
In this case study to obtain better results, compared to case – I and case – II the reactive power injections are varied between upper and lower limits (0 MVAR and 1.5 MVAR respectively) at all load buses. Summarized results of investment cost, operation cost and power loss are objective functions are shown in TABLE 3.

**TABLE 2: SUMMARIZED RESULTS OF GRAVER 6-BUS SYSTEM FOR CASE - II**

<table>
<thead>
<tr>
<th>Model/Objective Function</th>
<th>Investment Cost (Sx10^6)</th>
<th>Operation Cost ($/h)</th>
<th>Power Loss (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>110</td>
<td>614.4898</td>
<td>2.9448</td>
</tr>
<tr>
<td>CP LM</td>
<td>110</td>
<td>614.4778</td>
<td>2.9196</td>
</tr>
<tr>
<td>CC LM</td>
<td>100</td>
<td>606.8657</td>
<td>2.885</td>
</tr>
<tr>
<td>CZ LM</td>
<td>80</td>
<td>595.044</td>
<td>2.853</td>
</tr>
<tr>
<td>Mixed LM</td>
<td>100</td>
<td>612.316</td>
<td>2.9016</td>
</tr>
<tr>
<td>FFC</td>
<td>110</td>
<td>614.4864</td>
<td>2.929</td>
</tr>
<tr>
<td>FFC + CP LM</td>
<td>110</td>
<td>614.4891</td>
<td>2.929</td>
</tr>
<tr>
<td>FFC + CC LM</td>
<td>80</td>
<td>606.737</td>
<td>2.8722</td>
</tr>
<tr>
<td>FFC + CZ LM</td>
<td>80</td>
<td>594.9293</td>
<td>2.818</td>
</tr>
<tr>
<td>FFC + Mixed LM</td>
<td>110</td>
<td>612.2896</td>
<td>2.9011</td>
</tr>
</tbody>
</table>

Reactive power injected at all load buses are varied between minimum and maximum limits such that bus voltages are within limits and line loadings are also within their limits. Investment cost $80 million was in base case and also the optimum investment cost $80 million resulted in (a) base case, (b) constant power load model (CP LM), (c) constant current load model (CC LM), (d) mixed load model (mixed LM), (e) flat frequency control generator model (FFC), (f) combination of flat frequency control generator model and constant power load model (FFC+CP LM), (g) combination of flat frequency control generator model and constant current load model (FFC+CC LM), (h) combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM) and (i) combination of flat frequency control generator model and mixed load model (FFC+mixed LM).

Reactive power injected at all load buses are varied between minimum and maximum limits such that bus voltages are within limits and line loadings are also within their limits. Investment cost in base case was reduced to $80 million and the optimum investment cost also $80 million resulted in (a) base case, (b) constant power load model (CP LM), (c) constant current load model (CC LM), (d) constant impedance load model (CZ LM), (e) mixed load model (mixed LM), (f) flat frequency control generator model (FFC), (g) combination of flat frequency control generator model and constant power load model (FFC+CP LM), (h) combination of flat frequency control generator model and constant current load model (FFC+CC LM), (i) combination of flat frequency control generator model and constant load model (FFC+CZ LM) and (j) combination of flat frequency control generator model and mixed load model (FFC+mixed LM).

In addition of reactive power injection at all load buses, the power generations are varied by each generator within their minimum and maximum limits to maintain system healthy conditions. Operation cost as the objective function, optimum operation cost 595.8805 $/h resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

Likely the system was stabilized with respect to all equality and inequality conditions with different constraints. Power loss as the objective function, the optimum power loss 2.786 MW resulted in
combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

**TABLE 3: SUMMARIZED RESULTS OF GRAVER 6-BUS SYSTEM FOR CASE - III**

<table>
<thead>
<tr>
<th>Model/Objective Function</th>
<th>Investment Cost ($x10^6)</th>
<th>Operation Cost ($)</th>
<th>Power Loss (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>80</td>
<td>614.4523</td>
<td>2.824</td>
</tr>
<tr>
<td>CP LM</td>
<td>80</td>
<td>614.4493</td>
<td>2.8241</td>
</tr>
<tr>
<td>CC LM</td>
<td>80</td>
<td>607.0825</td>
<td>2.8133</td>
</tr>
<tr>
<td>CZ LM</td>
<td>80</td>
<td>595.8223</td>
<td>2.8002</td>
</tr>
<tr>
<td>Mixed LM</td>
<td>80</td>
<td>612.2788</td>
<td>2.8186</td>
</tr>
<tr>
<td>FFC</td>
<td>80</td>
<td>614.4613</td>
<td>2.8324</td>
</tr>
<tr>
<td>FFC + CP LM</td>
<td>80</td>
<td>614.4646</td>
<td>2.8324</td>
</tr>
<tr>
<td>FFC + CC LM</td>
<td>80</td>
<td>606.645</td>
<td>2.8127</td>
</tr>
<tr>
<td>FFC + CZ LM</td>
<td>80</td>
<td>595.8805</td>
<td>2.786</td>
</tr>
<tr>
<td>FFC + Mixed LM</td>
<td>80</td>
<td>612.3129</td>
<td>2.8247</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

This paper explored the possibility of applying AC-OPF based models to the TEP problem. Nine TEP models are presented in this paper. The formulation of each model is shown and discussed in detail. A validation process guarantees the resultant TEP plan is strictly AC feasible. The conclusions of this paper are: The AC model can be applied to model TEP problems. The solution of BA-based AC-TEP models is still challenging. By reformulation and relaxation, it is possible to solve the BA-based ACTEP problem and obtain an optimal solution.

**REFERENCES**


